ROBUST FLOW CONTROL OF SINGLE INPUT MULTIPLE OUTPUTS REGULATED RIVERS

By Xavier Litrico¹

ABSTRACT: Regulated rivers are used in many countries to sustain summer low flows and supply water for different users (drinking water, industries, irrigation, hydropower, salubrity). An upstream dam is used as storage, and the river is used as a channel to convey water-to-water users. Such systems are characterized by long and variable time delays between upstream and downstream points. This paper deals with the automatic control of such systems, where the action variable is the upstream discharge released by the dam, and the controlled variable is the downstream discharge of the river. As there are multiple outputs given by intermediate measurement points distributed along the river, the system is a cascade of single input, single output systems and is considered as a single input, multiple output one. A generic robust design synthesis is developed for internal model-based controllers. Simulations carried out on a nonlinear model of the Gimone River in the southwest of France show the improvement compared to present semimanual regulation.

INTRODUCTION

Regulated rivers are used in many countries to sustain summer low flows and supply water for different users (drinking water, industries, irrigation, hydropower, salubrity). An upstream dam is used as storage, and the river is used as a channel to convey water to water users. This use of natural channels prevents the conduct of important civil engineering work, as in the case of irrigation canals. On the other hand, such systems are difficult to manage, because there are usually only one control action variable located at the dam and a few measuring points along the river. The system is subject to large, nonmeasured perturbations (inflows, or withdrawals) and the dynamics of each reach (part of river between two measuring points) are strongly nonlinear. Therefore, it is difficult to determine the “optimal” water release at the upstream dam in order to maintain a target discharge downstream. These systems are generally managed manually but with low efficiency (Kosuth 1994). The application of automatic control techniques for real-time regulation of these open-channel hydraulic systems could lead to large water savings and better service.

In the following, the term “dam-river system” will be used for a regulated river with one dam at the upstream end and at least one measurement point at the downstream end of the river. Using approximated linear models, Papageorgiou and Messmer (1989) proposed design methods for a dam river with one reach but did not take into account robustness requirements, which are essential, especially for nonlinear systems controlled with linear regulators. Sawadogo (1992) proposed a method by inverse propagation for the open-loop control of dam-river systems with intermediate measurements. As the system considered is dominated by long, varying time delays, the robustness to time delay variations is very important. Kosuth (1994) studied robust stability by looking at the poles of the closed-loop system, but he did not end with a reliable tuning method for robust control.

Such a robust design approach is fairly recent for automatic control of irrigation systems. Only a few references have mentioned and evaluated model uncertainties (Corriga et al. 1989; Jreij 1997; Schuurmans 1997; Seatzu et al. 1998). Their approach was restricted to the control of canal systems, where

the elevation is controlled with intermediate gates, which is not the case for the regulated rivers considered. Significant nonlinearities are encountered in this latter case that make it compulsory to evaluate controller stability robustness. An analytic modeling method based on physical equations of open channel hydraulics is proposed, giving a nominal model and a bound on multiplicative uncertainties for a river reach. The approach already developed for single input single output (SISO) systems (Litrico and Georges 1999) is extended to the single input multiple outputs (SIMO) case, when there are multiple measuring points along the river.

SYSTEM DESCRIPTION

Presentation of the System

The irrigation system considered uses natural rivers to convey water released from an upstream dam to consumption places distributed along the river. The system is depicted in Fig. 1, with a dam and a river stretch, a measuring station at its downstream end and intermediate measurements. The examples in this paper will be developed for the case of a river with two measurements, but the methodology is applicable to any number of intermediate measurements.

The upstream discharge Q_{upstream} is the control action variable, also noted as u . Therefore, it is assumed that there is a local (slave) controller at the dam that acts on a gate such that the desired discharge is delivered. The discharge at the downstream end of the river $Q_{\text{downstream}}$ is the controlled variable, also noted as y_2 in the case of a river with two reaches; y_1 is an intermediate measured variable (a discharge).

Control Objectives and Constraints

The control objectives are mainly to keep the flow rate at the downstream end of the reach close to a reference flow rate (a target noted as z_c) defined for hygienic and ecological reasons. This has to be done while farmers and other users are withdrawing water from the river (pumping stations w_i). These not completely known withdrawals are considered as perturbations that have to be rejected by the feedback controller.

FIG. 1. Dam-River System with Intermediate Measurements

¹Res., Cemagref, UR IRMO, B.P. 5095, 34033 Montpellier Cedex 1, France. E-mail: xavier.litrico@cemagref.fr

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The water demand w_i can be predicted more or less accurately using weather forecast, soil-plant models, and a database of previous seasons. Water demand predictions are used in a feedforward controller. The variations in the demand are then considered as perturbations (due to unpredicted inflow or outflow) and have to be rejected by the feedback controller. There are saturation limits on the actuator: the discharge delivered by the barrage has to be within the bounds $[u_{\min}, u_{\max}]$ with $u_{\min} = 0.1 \text{ m}^3/\text{s}$ and $u_{\max} = 5 \text{ m}^3/\text{s}$. This is imposed on the system but not taken into account explicitly in the control design.

The main problem encountered in the automatic regulation of such systems is the robustness to varying time delays. As already stated by Papageorgiou and Messmer (1989), the dynamics of a river stretch are nonlinear and depend on the discharge. Such time delay variations can destabilize a linear controller designed without taking them into account.

Modeling of the System

Open channel flows are well described by Saint-Venant equations, which are two coupled nonlinear partial differential equations involving the discharge and the absolute water surface elevation. These equations require a lot of data (geometry of the channel, longitudinal profile, roughness coefficient) that are not always available in the case of rivers. This is why it is interesting to consider simplified models.

Under simplifying hypotheses Saint-Venant equations lead to the diffusive wave equation (Miller and Cunge 1975)

$$\frac{\partial Q}{\partial t} + C(Q, Z, x) \frac{\partial Q}{\partial x} - D(Q, Z, x) \frac{\partial^2 Q}{\partial x^2} = 0 \quad (1)$$

with $Q(x, t)$ the discharge $[\text{m}^3/\text{s}]$, $C(Q, Z, x)$ the celerity $[\text{m}/\text{s}]$, and $D(Q, Z, x)$ the diffusion $[\text{m}^2/\text{s}]$. The following form (obtained analytically for uniform large rectangular channels) is assumed for the celerity and diffusion coefficients as

$$C(Q) = \alpha_C Q^{\beta_C}, D(Q) = \alpha_D Q^{\beta_D} \quad (2)$$

This formulation has been validated on different geometries (simulated and real data) by identification of the four parameters $\alpha_C, \beta_C, \alpha_D, \beta_D$ (Litrico 2000).

Linearizing (1) around a reference discharge Q_0 leads to the Hayami equation, which can be analytically identified to a second order plus delay transfer function (Malaterre 1994) as

$$F(s) = \frac{\omega_n^2 e^{-s\tau}}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (3)$$

where τ = time delay; ω_n = natural frequency; ζ = damping; and s = Laplace variable. The analytical identification process enables us to express the coefficients of $F(s)$ as functions of physical parameters, as the reference discharge Q_0 .

Uncertainty Description

The uncertainties due to reference discharges in a bounded set are represented as an output multiplicative uncertainty. For each reach i , dynamic coefficients ($\alpha_{Ci}, \beta_{Ci}, \alpha_{Di}, \beta_{Di}$) are supposed to be known. With an evaluation of the extreme flow rates $[Q_{i\min}, Q_{i\max}]$, a bound on multiplicative uncertainty can be evaluated (Litrico 1999).

Each reach of the system can, therefore, be represented as in Fig. 2, where in input u is the upstream flow rate, the output y is the downstream flow rate, F_0 represents the function of nominal transfer of the reach, and E_m is the corresponding multiplicative uncertainty.

This multiplicative uncertainty E_m captures time delay as well as dynamics variations, which are due to the nonlinearity

of the system. For $Q_0 \in [Q_{\min}, Q_{\max}]$, the transfer function $F(s)$ is written as

$$F(s) = [1 + E_m(s)]F_0(s) \quad (4)$$

where

$$|E_m(j\omega)| \leq \max_{Q_0 \in [Q_{\min}, Q_{\max}]} \left| \frac{F(j\omega, Q_0)}{F_0(j\omega)} - 1 \right| \quad (5)$$

j = complex number such that $j^2 = -1$; $|a|$ represents the norm of a complex number $a \in \mathbb{C}$; and ω = frequency of a sinusoidal signal. $F_0(s)$ is the nominal model used to design the controller.

The uncertainty $E_m(s)$ is bounded by a rational function $W_m(s)$: $E_m(s) = W_m(s)\Delta(s)$, with $|\Delta(s)| < 1$, such that $|E_m(j\omega)| \leq |W_m(j\omega)| \forall \omega \in \mathbb{R}$ (Fig. 3). $\Delta(s)$ is the normalized uncertainty, and $W_m(s)$ is the frequency weighting function (rational and stable transfer function).

For a single reach represented by a monovariable transfer function, it is indifferent to represent the uncertainty in input

FIG. 2. Modeling of River Reach with Output Multiplicative Uncertainty

FIG. 3. Bound W_m on Output Multiplicative Uncertainty E_m

FIG. 4. Standard Representation of Dam-River System with Two Measurement Points

or in output. This is not true as soon as one deals with multivariable systems. Then, the uncertainty becomes structured (diagonal), as shown in Fig. 4, which gives the standard representation for a dam-river system. Introducing the intermediate inputs p_1, p_2 and outputs q_1, q_2 , the system can be represented in matrix form by the equations

$$(y_1, y_2, q_1, q_2)^T = M(u, p_1, p_2)^T \quad (6)$$

with

$$M = \begin{pmatrix} F_{10} & 1 & 0 \\ F_{10}F_{20} & F_{20} & 1 \\ W_{m1}F_{10} & 0 & 0 \\ W_{m2}F_{10}F_{20} & W_{m2}F_{20} & 0 \end{pmatrix} \quad (7)$$

and the uncertainty feedback is

$$(p_1, p_2)^T = \Delta(q_1, q_2)^T \quad (8)$$

with

$$\Delta = \begin{pmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{pmatrix} \quad (9)$$

The system M represents the nominal system augmented with weighting functions W_{mi} , $\Delta = \text{diag}(\Delta_i)$ represents the structured multiplicative uncertainties. As the frequency weighting functions $W_{mi}(s)$ are included in the nominal augmented system M , one gets $\|\Delta\|_\infty \leq 1$. The H_∞ norm of a transfer function $F(s)$ is defined as (Francis 1985)

$$\|F\|_\infty = \sup_\omega |F(j\omega)| \quad (10)$$

and for a transfer matrix $G(s)$ as

$$\|F\|_\infty = \sup_\omega \bar{\sigma}[G(j\omega)] \quad (11)$$

where $\bar{\sigma}(A)$ denotes the larger singular value of matrix A .

As the uncertainty Δ is structured, the stability robustness of the controlled system can be evaluated using the structured singular value μ (Doyle 1982). This structured singular value extends the results obtained in monovariable robust control (which uses the larger singular value) to multivariable systems with structured uncertainties. In particular, a controlled system is stable with respect to structured uncertainties $\Delta \in \Delta$ if and only if

$$\mu_\Delta[M(j\omega)] \leq 1 \quad \forall \omega \in \mathbb{R} \quad (12)$$

This robust stability condition is used in the design of the controller to determine the values of the filters coefficients such that the feedback system is robust stable.

CONTROLLER DESIGN

Internal Model Control Based Architecture

Description

The internal model control (IMC) based architecture, inspired from a controller designed by Compagnie d'Aménagement des Coteaux de Gascogne (CACG) currently implemented in the field, uses the models of each subsystem. It enables an independent estimation of intermediate withdrawals in each reach (Fig. 5). Therefore, the controller does not react more than once to the same withdrawal. The open-loop command u_{ol} is obtained by pseudoinversion of nominal transfer functions, leading to stable transfer functions. These pseudoinverses include the time delay, as it is assumed that the withdrawals can be predicted in advance in order to be able to compensate for them. The procedure used to design stable pseudoinverses of the transfer functions is not detailed here,

FIG. 5. IMC Controller Architecture for SIMO Dam-River Systems

because the main focus of the paper is the design of a robust feedback controller.

The closed-loop command is obtained by comparing the measures on the system (denoted y_1 and y_2) to the outputs of the internal model (transfer functions denoted F_{10} and F_{20}), taking into account the predicted withdrawals w_{p1} and w_{p2} . The difference represents the internal model estimation of unpredicted withdrawals w_{u1} and w_{u2} . This estimation is filtered by the filters f_1 and f_2 . Their outputs are summed to give the closed-loop command u_{cl} . The filters are designed to compensate for the discrepancies between the predicted values w_{pi} and the actual ones. If the nominal model was ideally a perfect description of the system, and the predicted values w_{pi} were ideally accurate, the effect of the unpredicted withdrawals would be exactly accounted for by the control scheme.

The choice of controller parameter then reduces to the choice of robustness filters for each reach. Filters f_i are chosen of first order, so there is only one parameter for each reach, the filter time constant.

Using the standard representation of Fig. 4 with the IMG-SIMO controller, the following relations are obtained:

$$(q_1, q_2)^T = M_{\text{IMC}}(p_1, p_2)^T \quad (13)$$

with

$$M_{\text{IMC}} = - \begin{pmatrix} W_{m1}F_{10}f_1 & W_{m1}F_{10}f_2 \\ W_{m2}F_{20}(f_1F_{10} - 1) & W_{m2}F_{20}F_{10}f_2 \end{pmatrix} \quad (14)$$

With $F_{i0} = B_{i0}/A_{i0}$, and $f_i = f_{i\text{num}}/f_{i\text{den}}$, the characteristic polynomial (denominator of the closed-loop system) is

$$P_{\text{IMC}} = A_{10}A_{20}f_{1\text{den}}f_{2\text{den}} \quad (15)$$

The nominal closed-loop system is stable if and only if the characteristic polynomial is stable. Therefore, the nominal stability of the closed-loop system is guaranteed if and only if

- The nominal transfer functions F_{i0} are stable
- The filters f_i are stable

The first condition is always verified for a dam-river system, and with stable filters, the nominal stability of the IMC architecture is ensured. This is one major advantage of the IMC-SIMO architecture, to guarantee a priori nominal stability, without proceeding by trial and error.

Robust Stability Condition

A theorem given by (Doyle 1982) provides a necessary and sufficient condition of robust stability under structured uncer-

tainty using the structured singular value. Considering the system with a multivariable approach, the closed-loop system has to satisfy the following robust stability condition.

The IMC-SIMO command architecture of Fig. 5 is stable for every structured model error $\Delta \in \mathbf{\Delta}$ such that $\|\Delta\|_\infty \leq 1$ if and only if

$$\mu_\Delta[M_{\text{IMC}}(j\omega)] < 1 \quad \forall \omega \in \mathbb{R} \quad (16)$$

with M_{IMC} given by (14) and Δ by (9). $\mathbf{\Delta}$ represents the set of matrices Δ sharing the same structure. This latter condition will be used to check robust stability of the closed-loop system for the IMC-SIMO controller design. The controller design uses a parameterization for the filters in order to ensure robust stability of the closed loop system.

Controller Parameterization

The controller parameterization is done as a function of the main dynamic uncertainty—the time-delay variation for each reach. The filter coefficient determination is based on the following remark: the more the uncertainty on the time-delay is important, the more the measure must be filtered in order for the closed-loop to be stable.

Suppose the dynamic uncertainties are negligible, then a perfect knowledge of the time delay would enable almost no filtering, as the withdrawal estimation by the IMC controller would be perfect. However, as the time delay is variable, the filter time constant is linked to the uncertainty on the time delay between the dam and the considered measurement point.

The considered filters are of the first order in the form

$$f(s) = \frac{1}{1 + sT} \quad (17)$$

where T = filter time constant.

The step-response of a first-order filter $f(s)$ is given by the equations: $f(t) = 0$ for $t \leq 0$ and $f(t) = 1 - e^{-t/T}$ for $t > 0$. Given x a positive real between 0 and 1, one may compute the rise time at 100% as a function of the time constant T

$$t_{100\%} = \ln\left(\frac{1}{1-x}\right) T = -\ln(1-x)T \quad (18)$$

where $\ln$ stands for the natural logarithm. For example, the rise time at 90% ($x = 0.9$) is given by $t_{90} = 2.3T$.

For each reach i , $\delta\tau_i$ is the maximum uncertainty on the time delay τ_{i0} . This uncertainty is given by

$$\delta\tau_i = \max(|\tau_{i\min} - \tau_{i0}|, |\tau_{i\max} - \tau_{i0}|) \quad (19)$$

where $\tau_{i\min}$ and $\tau_{i\max}$ are the maximum and minimum time delays, respectively, obtained for the minimum and maximum flow rates $Q_{i\max}$ and $Q_{i\min}$; and τ_{i0} is the nominal time-delay of the reach i .

Let $\Delta\tau_i$ be the uncertainty on the total time delay from the dam to the downstream end of the reach i

$$\Delta\tau_i = \sum_{k=1}^i \delta\tau_k \quad (20)$$

Filter coefficients for each reach i are then defined with respect to the maximum uncertainty on the total time delay $\Delta\tau_i$, specifying the desired percentage x such that the filter reacts at 100% in a rise time equal to $\Delta\tau_i$. This means that the parameter x is used as a synthesis parameter in order to get more or less important filtering with a rise time linked to the uncertainty on the total time delay. Then, a withdrawal detected by the internal model controller will be more filtered if this “withdrawal” may be due to a bad estimation of the time delay of the system.

The following relation is obtained by

$$T_i = -\frac{\Delta\tau_i}{\ln(1-x)} \quad (21)$$

where T_i the time constant of the filter for reach i .

Taking into account the uncertainty on total time delay imposes the following relations on T_i

$$T_1 < T_2 < \dots < T_n \quad (22)$$

This condition is useful to guarantee a quicker reaction to perturbations detected in the upstream reaches than for perturbations detected in downstream ones.

Robust Synthesis

This parameterization has the advantage to simplify the robust design procedure. When x equals zero, i.e., when the filters have an infinite time constant, the closed loop stability is ensured (as the system is in fact in open loop). When x increases, the reaction of the filters will become greater for a similar withdrawal, and the robustness of the closed loop will decrease. The robust design procedure consists in determining the greatest value of x , such that the robust stability condition involving the structured singular value is verified. The value of x is determined by a bisection algorithm until the robust stability condition $\mu_\Delta(M_{\text{IMC}}) < 1$ is satisfied.

APPLICATION AND DISCUSSION

The procedure is applied to the Gimone River, located in the southwest of France and managed by the CACG. This 116-km long river is fed by the Lunax Dam and has two measurement points situated at Gimont (43.7 km) and Castelferrus (116.1 km). Therefore, the river has two reaches of lengths 43.7 km and 72.4 km, representing average delays of respectively 16 h and 31 h for the nominal discharges given in Table 1.

Simulations are carried out on a simplified nonlinear model of the Gimone, obtained by identification from real data. Parameters are given in Table 2.

Controller Synthesis

Fig. 6 displays the values of μ_Δ as a function of percentage x for the IMC-SIMO controller for the Gimone River. One may observe that the closed loop stability margin decreases as the percentage x increases, which is coherent with the theory. With a value of x slightly greater than 0.6, the closed-loop is robust stable, as $\mu_\Delta(M_{\text{IMC}}) < 1$.

The parameter x such that $\mu < 1$ is determined automatically

TABLE 1. Nominal and Extreme Discharge Values, Gimone Reaches

Discharge	Reach 1	Reach 2
$Q_{\min}$ [m^3s^{-1}]	0.30	0.20
Q_0 [m^3s^{-1}]	0.67	0.50
$Q_{\max}$ [m^3s^{-1}]	2.50	2.00

TABLE 2. Nonlinear and Linear Models Parameter Values, Gimone Reaches

Parameters	Reach 1	Reach 2
α_c	0.5809	0.6953
β_c	0.4622	0.3697
α_D	626.76	373.74
β_D	0.4947	0.8694
C_0 [ms^{-1}]	0.4842	0.5375
D_0 [m^2s^{-1}]	515.81	204.05
τ [s]	$5.8190 \cdot 10^4$	$1.1154 \cdot 10^5$
ω_n [s^{-1}]	$5.631 \cdot 10^{-5}$	$7.607 \cdot 10^{-5}$
ζ	0.9027	0.8805

FIG. 6. Structured Singular Value μ_{Δ} as Function of Parameter x

by a bisection algorithm, starting from $x_{\min} = 0$ and $x_{\max} = 1$. This method leads to the value

$$x = 0.619 \quad (23)$$

giving the following filters time constant:

$$T_1 = 42370s = 11h \text{ and } 46min$$

$$T_2 = 98347s = 27h \text{ and } 19min$$

The continuous-time controller designed is sampled with a sampling time equal to 3 h, the value used in the field.

Simulations

Simulations done on typical scenarios showed the stability and performance of the controller for different flow rates inside the considered domain (Litrico 1999). The controller is also tested, using data provided by CACG for the 1997 season, to compare it with the current semimanual controller implemented in the field. As the withdrawals are not measured, the upstream and downstream measured discharges are used to identify the withdrawals in each reach.

1997 Season Data

With hourly flow rate data measured on the Gimone for the 1997 season, withdrawals are identified using the simplified nonlinear dynamical model, assuming there is only one withdrawal point in each reach. Negative withdrawals correspond to inflows or rains. For the scenario simulating the 1997 season, withdrawals w are supposed to be known at 75%. The assumption is that predictions are underestimating the demand of 1/4, which means that if the real demand is 1 m³/s, the estimation will predict a demand of 0.75 m³/s (value used by the feedforward controller). The other 0.25 m³/s will be considered as unpredicted withdrawals. Such an assumption is rather pessimistic, as the predicted value is never equal to the real one. The choice of the values 75 to 25% was proposed by the managers of the CACG, because they could predict about 3/4th of the demand. Therefore, without a feedback controller, the output discharge y_2 would never be equal to the desired output discharge z_c . The real and predicted withdrawals used in the simulation are shown in Fig. 7.

The feedforward controller uses the predicted withdrawals to compute the open-loop control, whereas the feedback controller has to cope with the unpredicted withdrawals and the modeling errors.

Simulation Results (1997 Season)

The beginning of the simulation corresponds to July 20, and the end to August 27. The overall simulation represents 900 h or 37 days. The period is representative of a high irrigation level for the beginning, and rains at the end of the period leading to a decrease in the withdrawals. The targeted downstream flow rate changes according to the minimum discharge requirements for hygienic reasons and a flow rate requirement to supply hydroelectric plants downstream. Results of simulation are depicted in Fig. 8. The controller is tested on the simplified nonlinear model of the river with parameters given in Table 2. This simplified nonlinear model solves the diffusive wave equation (1) with the Muskingum-Cunge numerical discretization scheme (Cunge 1969).

The controller is stable around minimum and maximum discharges, and can use forecasted events such as predicted withdrawals in one reach in order to minimize their effect on the controlled downstream discharge. Because of the strong nonlinearity of the system and the conservative representation of the model uncertainties, the control is rather slow in order to ensure robust stability. This explains the fact that the downstream discharge nearly goes to zero during a few hours at the

FIG. 7. Real (—) and Predicted (---) Withdrawals, Reaches 1 and 2 of Gimone Data

FIG. 8. Command u , Outputs y_1 (---), y_2 (—), and Reference z_c (····), IMC-SIMO Controller, Gimone 1997 Data

beginning of the irrigation (time $t = 95$ h), as the demand raises very quickly and is not well estimated. The shortage of water is difficult for which to compensate as the time delay of the system is very long. The presence of such a long time delay limits the performance of the controlled system. It is not possible to compensate exactly for large deviations in discharge downstream of the system if they have not been properly predicted. This type of critical period is also encountered by the manager, especially at the beginning of the season and after rains, because the irrigation demand varies quickly and in an unpredictable way.

In the season, the controller behaves well and efficiently maintains the discharge, following the increase in the targeted flow, and quickly responds to the rains occurring in reach 2 at time $t = 690$ h and $t = 880$ h. Compared to the semimanual control currently implemented in the field, this new controller saves as much as 30% of the volume of the dam for the simulation on the whole season (about 2,000,000 m³). These water savings may not be as important in reality, because the manager tends to deliver more water than necessary to limit the shortage of water downstream. However, the proposed controller gets good performance from the feedback controlled system. The feedforward controller can be optimized to be more efficient, but this is beyond the scope of this paper. Some work is also required on the prediction of the water demand.

CONCLUSION

A solution is given to the problem of designing a robust architecture for controlling dam-river systems with intermediate measurement points. It uses the internal model control design approach. The IMC procedure is adapted to the SIMO case, and the IMC controller is parameterized according to the main uncertainty (time-delay), leaving only one design parameter. This parameterization enables an automatic robust design method, as the value of the parameter x such that the robust stability condition is verified and computed automatically.

The proposed IMC-SIMO controller has been in a joint research collaboration with CACG to improve the real-time control algorithms for dam-river systems. This controller is planned to be implemented in the field for the irrigation season of year 2001.

This type of controller can be extended to dam-river systems with more than one dam, but as the system develops multiple inputs multiple outputs, multivariable controller design techniques lead to better performance (Litrico 1999). Future research will deal with the application of other robust design techniques such as H_∞ control to the same systems in order to compare it to the controller presented here.

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NOTATIONS

The following symbols are used in this paper:

- C = celerity [ms⁻¹];
 D = diffusion [m²s⁻¹];
 $E_m(s)$ = multiplicative model error;
 $F_{i0}(s)$ = nominal transfer function for reach i ;
 $f_i = f_{inum}/f_{iden}$ = first order filter for reach i , IMC-SUMO architecture;
 M = nominal augmented system for robustness analysis;
 Q = discharge [m³s⁻¹];
 s = Laplace variable;
 u = command variable (upstream discharge);
 $W_m(s)$ = rational weighting function;
 w_{pi} = predicted withdrawals in reach i ;
 w_{ui} = unpredicted withdrawals in reach i ;
 x = parameter of the IMC-SIMO controller;
 y_i = measured output of reach i (discharge);
 z_c = reference discharge at the downstream end;
 $(\alpha_c, \beta_c, \alpha_D, \beta_D)$ = parameters for nonlinear model;
 $\Delta_i(s)$ = model uncertainty for reach i ;
 μ_Δ = structured singular value relative to the structure Δ ;
 $\bar{\sigma}$ = maximum singular value; and
 ω_n, ζ, τ = transfer function parameters for second order plus delay model.