Nonlinear identification of an irrigation system

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Abstract - The paper addresses the problem of the identification of an irrigation system in presence of not completely known withdrawals. The approach is based on a time-implicit finite-dimensional model of simplified equations governing open-channel hydraulics. A two stage iterative method is proposed to identify the dynamics and the withdrawals. In the first stage only the parameters governing the withdrawals are estimated, and in the second stage, the estimated withdrawals are then used to estimate the parameters of the dynamics. The simulation studies performed on simulated as well as on real data show the effectiveness of the identification method.

Key words : Identification, nonlinear system, dam-river system, sequential optimization, Diffusive Wave equation

1. INTRODUCTION

1.1. A brief review
System identification is often an important step in the synthesis of a control law. The procedure is well understood and analyzed for parametric models used by automaticians [9]. It is not always the case when people use knowledge based models, because the identifiability is often difficult to establish, and because the identification is traditionally done in another way. In the case of open channels hydraulic systems, identification is usually done by trial and errors and simulations. We are aware of few other methods used for identification in hydraulic systems ([12], [19], [18]). With an explicit model, Georges [5] showed the observability and identifiability with two measurements in a reach (discharge and/or water level). For implicit models, global identifiability is still an open problem. For MIMO systems, in the case when there are more than one input, multistage methods are used and applied to different substructures [9]. These methods are classified in two types, namely sequential optimization and parallel optimization methods [2].
We propose in the paper a two-stage sequential optimization method to identify the dynamics and the withdrawals of a dam-river system used for irrigation purposes.

1.2. Presentation of the system
In the South of France, the irrigation system managed by the Compagnie d’Aménagement des Côteaux de Gascogne (CACG) uses natural rivers to convey water released from the upstream dam to consumption places. It is an on-demand system, where farmers pump the water available in the river when they need it. The main objective of the regulation is to satisfy, in spite of uncertainties, the water demand at each pumping station while guaranteeing a minimum discharge all along the canal and spending a minimum water volume from the upstream reservoir.
We will consider the simplified system depicted in Figure 1: a dam and one river reach with a measuring station at its downstream end.

Figure 1: Simplified dam-river system

If we consider this system as a SISO system, with the upstream discharge as the input and downstream discharge as the output, its main feature is to have varying time delays. When the discharge is not varying too much, this system can be modeled by a rational transfer function of second order with delay ([14], [16], [4]). The system is currently regulated with pole placement controllers, which become unstable when the time delay changes [7].
Sawadogo [18] proposes a modification of Hayami linear model to take into account the variation of time delay with the discharge. He uses there an adaptive approach, as the Hayami coefficient $\Theta$ is modified according to previous measures. The approach of this paper is to propose an alternative nonlinear modeling that includes the variation of time delay, using a discretized model of simplified equations of open-channels hydraulics.

2. MODEL STRUCTURE

2.1. Numerical model
The chosen model is a nonlinear model based on the discretization of partial differential equations representing the physics of the phenomenon. Open channel hydraulics are well represented by St-Venant equations [1]:

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\[
\begin{align*}
&\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial x} = q \\
&\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/S)}{\partial x} + Sg \frac{\partial z}{\partial x} = -SgJ + qV
\end{align*}
\]

Q is the discharge (m³/s) across section S, q the lateral discharge (m³/s), S the wetted area (m²), z the absolute water surface elevation (m), J the friction slope, V the mean velocity (m/s) in section S, and g the gravitational acceleration (m/s²).

If q = 0 and we neglect inertia terms \(\left(\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/S)}{\partial x}\right)\) in front of \((Sg \frac{\partial z}{\partial x})\), we get:

\[
\frac{1}{Sg} \frac{\partial z}{\partial t} + \frac{\partial Q}{\partial x} = 0
\]

\[
\frac{\partial z}{\partial x} = -J
\]

Differentiating the first equation with respect to x and the second with respect to t, and after some rearranging, we get a nonlinear partial differential equation, called the Diffusive Wave equation [11]:

\[
\frac{\partial Q}{\partial t} + C(Q,x)\frac{\partial Q}{\partial x} - D(Q,x)\frac{\partial^2 Q}{\partial x^2} = 0
\]

with \(Q(x,t)\) the discharge (m³/s), \(C(Q,x)\) the celerity (m/s), and \(D(Q,x)\) the diffusion (m²/s).

The linear equation obtained with the hypothesis that C and D are constant is called Hayami equation. This hypothesis is valid when the discharge is not varying too much [10].

In general cases, it is difficult to have analytical expressions of C and D, but if we assume uniform geometry, and quasi-uniform flow, we can express C and D in function of Q and parameters describing the geometry of the river. For a rectangular river of width L, slope I, and uniform depth y, we have:

\[
C = \frac{5Q}{3Ly}, \quad D = \frac{Q}{2LI}
\]

Then D is proportional to Q, while C is a power function of Q in an increasing way, as y can be expressed in function of Q using Manning-Strickler formula [1] (assuming y is equal to the hydraulic radius).

Taking the variables \(\Theta = \frac{1}{C}\) (hydraulic time-lag in s/m) and \(Z = \frac{C}{4D}\) (amplification in m³), Trouvat [19] proposed to express \(\Theta\) and \(Z\) as power functions of Q.

We make the same assumption with C and D, and we identify the four parameters \(\alpha_c, \beta_c, \alpha_d, \beta_d\) (all positive), such that \(C(Q) = \alpha_c Q^{\beta_c}\) and \(D(Q) = \alpha_d Q^{\beta_d}\).

The discretization of the Diffusive Wave equation with an implicit scheme (Crank-Nicholson scheme [13]) is problematic, because it needs a downstream boundary condition in terms of discharge, which is not physical. One possibility mentioned in literature ([11], [15], [8]) is to solve the Kinematic Wave equation with a particular scheme, that leads to a routing method called Muskingum-Cunge method [3].

The Kinematic Wave equation represents a simple (still nonlinear) transfer, without diffusion:

\[
\frac{\partial Q}{\partial t} + C(Q,x,t)\frac{\partial Q}{\partial x} = 0
\]

This equation is discretized with a two points implicit scheme:

Figure 2: Two points implicit discretization scheme

The partial derivatives of Q are written as follows:

\[
\frac{\partial Q}{\partial t} = \frac{1}{\Delta t} \left[ Q^{k+1}_i - Q^k_i \right] + (1-\theta) \frac{Q^{k+1}_{i+1} - Q^{k+1}_i}{\Delta t}
\]

\[
\frac{\partial Q}{\partial x} = \frac{1}{2\Delta x} \left[ Q^{k+1}_i - Q^{k+1}_{i+1} - Q^k_i + Q^k_{i+1} \right]
\]

which leads to an equation:

\[
A_{k+1}^i Q^{k+1}_i + A_{k+1}^{i+1}_i Q^{k+1}_{i+1} = B_1^i_1 Q^k_i + B_{2,1}^i_1 Q^k_{i+1}
\]

with \(A_{k+1}^i_1 = \frac{\Theta}{\Delta t} + \frac{C^{k+1}_i}{2\Delta x}, \quad A_{k+1}^{i+1}_1 = \frac{\Theta}{\Delta t} + \frac{C^{k+1}_{i+1}}{2\Delta x}\),

\[
B_1^i_1 = \frac{\Theta}{\Delta t} \frac{C^k_i}{2\Delta x}, \quad B_{2,1}^i_1 = \frac{1-\Theta}{\Delta t} \frac{C^k_i}{2\Delta x}
\]

\(C^{k+1}_i\) is calculated as:

\[C^{k+1}_i = C(Q_i), \quad \text{with} \quad Q_a = \frac{Q^k_i + Q^k_{i+1} + Q^{k+1}_i}{3}\]

This approximated expression leads to good numerical results, and simplifies the computation ([8], [15]).

With this two space steps discretization scheme, there is no need for a downstream boundary condition, and the upstream boundary condition is given by the discharge \(Q^{k+1}_i\).

The study of the discretized equation shows that the scheme is consistent to the first order if \(\Theta < 0.5\). The numerical diffusion introduced is equal to

\[
D_n = -C \Delta x \left( \Theta - \frac{1}{2} \right), \quad \text{as the second order term is equal to}
\]

\[
e = C \Delta x \left( \Theta - \frac{1}{2} \right) \frac{\partial^2 Q}{\partial x^2} + O(\Delta x^3)
\]

Therefore, if the stability conditions are fulfilled, one can solve by this means a Diffusive Wave equation, if one chooses:

\[
\Theta = \frac{1}{2} \frac{D}{C \Delta x}
\]

This enables to take into account the hydraulic diffusion phenomenon by the means of a numerical trick. Compared to a complete Diffusive Wave solution, the advantage is that the system to solve is simpler, requires less computing.
time, and easily enables the introduction of withdrawals that can occur in the river.

The equation of Kinematic Wave with diffusion is then given by:
\[ A_{11} Q_{k+1}^{k+1} + A_{21} Q_{k+1}^{k+1} = B_{11} Q_{k}^{k} + B_{21} Q_{k+1}^{k} \]
with \( A_{11} = \frac{1}{2\Delta t} - \frac{D_{11}^{k+1}}{\Delta x \Delta t C_{11}^{k+1}} - \frac{2\Delta x}{C_{11}^{k+1}} \),
\( A_{21} = \frac{1}{2\Delta t} + \frac{D_{21}^{k+1}}{\Delta x \Delta t C_{21}^{k+1}} + \frac{2\Delta x}{C_{21}^{k+1}} \),
\( B_{11} = \frac{1}{2\Delta t} - \frac{D_{11}^{k+1}}{\Delta x \Delta t C_{11}^{k+1}} + \frac{C_{11}^{k+1}}{2\Delta x} \), and
\( B_{21} = \frac{1}{2\Delta t} + \frac{D_{21}^{k+1}}{\Delta x \Delta t C_{21}^{k+1}} - \frac{C_{21}^{k+1}}{2\Delta x} \),
where \( D_{11}^{k+1} = D(Q_{o}) \).

With a state vector \( X = (Q_{0}, Q_{1}, ..., Q_{n})^{T} \), we get an equation:
\( A_{1} X^{k+1} = A_{2} X^{k} + B_{1} U^{k} \), with \( A_{1} \) and \( A_{2} \) depending on \( X^{k} \) and \( X^{k+1} \).

The input \( U^{k} \) is given by \( Q_{1}^{k+1} \), and the calculated output \( y_{c}^{k} \) is given by \( y_{c}^{k} = C_{1} X^{k} \), with \( C_{1} = [0 \ 0 \ ... \ 0 \ 1] \).

The withdrawals \( W_{p} \) are introduced by writing the conservation equation between two sections:
\( Q_{n} = Q_{n} + W_{p} \), \( W_{p} < 0 \) if withdrawal, \( > 0 \) if additional inflow.

As \( W_{p} \) depends on the parameters defined for the perturbation, we write it as a function of \( p \), vector of parameters.

The equation is finally:
\( h(X^{k+1}, X^{k}, d) = f(X^{k}, U^{k}, W^{k}(p), d) \)
\( y_{c}^{k}(d, p) = C_{1} X^{k} \)
with \( d \) representing the dynamics of the system:
\( d = [\alpha_{e} \beta_{e} \alpha_{o} \beta_{o}]^{T} \).

As the location of most of the pumping stations is known, the withdrawals are written as:
\( W^{k}(p) = W_{p}^{e} \lambda_{e}^{k}(p) \),
with \( W_{p}^{e} \) being the vector of maximum discharge pumped along the river (supposed to be known), and \( \lambda_{e}^{k}(p) \) being an irrigation coefficient representing the percentage of the maximum withdrawal effectively pumped. In \( W_{p}^{e} \), the individual pumping stations are combined into one station every 10 km. If the location of the withdrawals is not known, we propose to assume that they are uniformly distributed along the river.

2.2. Accuracy of the model
The Kinematic Wave model shows very good results compared to complete Saint-Venant model (see 4.1). It represents well the non-linearity of the system due to the variation of time delay with the discharge.

In Figure 3, the output of our Kinematic Wave model (where the time delay decreases as the discharge increases) is compared to Hayami linear model (where the time delay is constant). The error in the time of arrival the peak discharge is of about two hours (here for a 20 km long river).

3. IDENTIFICATION ALGORITHM

3.1. Identifiability
In general, when perturbations are equivalent to inputs, they are considered as secondary inputs [17]. The hypothesis is that the perturbations and the inputs are « independent ».

Nonetheless, it is possible to try to identify a model with perturbations, when the perturbations are considered as parameters of the model.

In our case, the withdrawals, if they are considered constant, can be identified when the initial state is a steady state. Then, it is easy to identify them by subtraction.

But if the initial state is not a steady state, the system with withdrawals is not identifiable, as we show it with a counter-example:

if the initial state is not a steady state, the error of identification \( e_{i} \) will be non zero, even in perfect case:
\( e_{i} \geq \sum_{k=1}^{T} \left| y_{m}^{k} - y_{c}^{k}(d, p) \right|^{2} \), with \( T \) the time delay of the system.

For example, suppose that the real model is given by:
\( d = d^{*} \) and \( W_{p} = 0 \) (or \( p = 0 \)).

Then, it will be possible to find an error lower than \( e_{i} \) by introducing a withdrawal \( W_{p} \neq 0 \). See Figure 4 where the input is constant, but, as the initial state is not a steady state, the output is different from the input at \( t = 0 \).

![Figure 3: Comparison of the output of the linear model (Hayami) with a nonlinear model (Kinematic Wave)](image)

![Figure 4: Example of non-identifiability of the system with withdrawals](image)
As the system with withdrawals is not identifiable when we do not know the initial state (which is the case in general), we chose to reduce the influence of the bias introduced by the initial steady state hypothesis by computing the quadratic criteria J only for k = T. T can be estimated by the formula T = Θ * X (with Θ = 1/C, and X = length of the river reach).

The optimization tested on the five parameters (d, p) proved to be ill conditioned. For such large scale problems, the use of a relaxation algorithm enables to solve lower scale problems which are better numerically conditioned. Here, instead of a globally non identifiable problem, we have to solve two identifiable sub-problems.

The identifiability of our implicit model is difficult to establish in general. With an explicit model for similar systems, Georges has proved the observability and identifiability [5] with two measurement in a reach. For implicit models, if analytical solutions are not available, a numerical result can still be obtained on the rank of the Jacobian matrix.

3.2. Sequential optimization with relaxation

We have the following representation of our system:

\[ \begin{align*}
    h(x^{k+1}, x^k, d) &= g(x^k, U^k, W^k(p), d)
    \\
    y_p^k(d, p) &= C_i x^k
\end{align*} \]

with \( W_p^k(p) = W_p^k(p) \), \( W_p^k(p) \) is a \( N_x \) vector giving the maximum withdrawal at each section (supposed to be known), and \( \lambda^k(p) \) is an irrigation coefficient, depending on parameter p.

The identification procedure will apply to \( d \) vector giving the dynamics of the system, and p, parameter for the withdrawals.

We have tested two possibilities:

\[ \lambda^k(p) = \lambda_c \text{ constant over time, or} \]
\[ \lambda^k(p) = \lambda^k(p_1...p_n) \text{ periodic law linear wise} \]

depending on two parameters, \( p_1 = \text{maximum irrigation coefficient, } p_2 = \text{minimum irrigation coefficient.} \)

We work on the basis of given measurement time-series \( \{y_m^k, k=1,...,N_m\} \), and input time-series \( \{u_m^k, k=1,...,N_m\} \). We consider identification schemes based on output error minimization.

We note \( J(d, p) = \sum_{k=1}^{N_m} \left( y_m^k - y_p^k(d, p) \right)^2 \)

\( y_m^k \) = measured output (downstream discharge).

\( y_p^k(d, p) \) = calculated output (downstream discharge).

**Identification algorithm:**

**Initialization:** choose \( (d_0, p_0) \). The initial condition \( X_0 \) is supposed to be steady state.

At iteration \( i: \)

step \( i: d, \leftarrow d_{i-1} \)

1) Identification of the withdrawals \( W^k(p), k=1,...,N_r \)

\[ \min_{d} J(d, p) \]

subject to:

\[ \begin{align*}
    h(x^{k+1}, x^k, d) &= g(x^k, U^k, W^k(p), d) \\
    y_p^k(d, p) &= C_i x^k
\end{align*} \]

with \( W^k(p) = W_p^k(p) \)

\[ \Rightarrow p_{i+1} = \text{Argmin} \left[ J(d, p) \right] \text{ and } W^k(p), k=1,...,N_r. \]

2) Identification of the dynamics

\[ \min_{d} J(d, p \_i) \]

subject to:

\[ \begin{align*}
    h(x^{k+1}, x^k, d) &= g(x^k, U^k, W^k(p), d) \\
    y_p^k(d, p) &= C_i x^k
\end{align*} \]

\[ \Rightarrow \lambda^i(p) = \text{Argmin} \left[ J(d, p \_i) \right] \]

\[ e_1 = \| p_{i} - p \|^2, e_2 = \| d_{i} - d \|^2 \]

\( d_i \leftarrow d_{i-1}, p_i \leftarrow p_{i-1} \) and go to 1.

Stop iterations when \( e_1 < e_1 \) or \( e_2 < e_2 \), with \( e_1 = 10^{-3}, e_2 = 10^{-2} \).

The minimization steps are done using the « leastsq » function in the Optimization Toolbox of MATLAB [6], which uses a Levenberg-Marquardt algorithm.

**Remark:** This relaxation method does not lead to a global minimum; first of all, there is no guarantee that it will converge, and secondly, as many optimization methods, it only gives a local minimum. Then we stopped the identification when the number of iterations becomes too big (>200). It is then better to begin again the identification with new initial values.

4. RESULTS OF IDENTIFICATION

4.1. Identification with simulated data

To validate our approach, the identification process is tested on simulated data. A river is simulated with an hydraulic simulation model based on complete Saint-Venant equations (SIC). The simulated river has the following characteristics: length \( X = 20 \) km; geometry: rectangular; width \( L = 10 \) m; Strickler coefficient \( K = 25 \); slope \( I = 0.0015 \); Withdrawals: 3 pumping stations, location \( 5, 10 \) and \( 15 \) km, with maximum withdrawals resp. -0.25, -0.30, and -0.45 m³/s.

The identification is done on three different situations: no withdrawals \( \lambda = 0 \), constant withdrawals \( \lambda = 1 \) and variable withdrawals \( \lambda \) changing from 0.5 to 1. The identification is done on the basis of a constant withdrawal. Those 3 situations are simulated with 3 different mean discharges \( 1, 2 \) and \( 3 \) m³/s.

Results of simulation show a good identification when the withdrawals were constant, but not very good results for variable withdrawals (which was expected). It is therefore very important to carefully select the data used for identification.

4.2. Identification with real data

The data are provided by the CACG, which manages the rivers in the Côteaux de Gasconne (Pyrénées, France). The river is the Baise, more precisely its second reach, which is 21.5 km long. The second reach of the Baise has two
pumping stations (individual withdrawals are aggregated each 10km). The function \( W_o \) is depicted in Figure 5. The identification is done on a 5-day period (120 hours). We compare the identification with constant withdrawals, and periodic withdrawals.

Before beginning the identification process, the first step is to select an appropriate period for identification, such a period needs to have « significant » hydraulic events, and not to be disturbed by precipitations. Then, the hypothesis made on the withdrawals will be acceptable, and the identification will not be significantly biased.

\[ \text{Figure 5: Maximum withdrawals } W_o, \text{ Baïse reach 2} \]

Figure 6 shows the results of identification for the second reach of the Baïse for a 5-day period (9 to 14 August 1994).

\[ \text{Figure 6: Result of identification: input (dashed), measured output (solid) and model output (dotted)} \]

We observed a strong variability of the diffusion parameters \((\alpha_0 \text{ and } \beta_0)\). In fact, the criteria is not very sensitive to fluctuations of these parameters, and, as shown in 4.1., the error made on the identification of withdrawals may give non-physical values of \( \beta_0 \). To reduce the variability of the parameters, we can impose a relation between two parameters, and identify only three dynamic parameters instead of four. As the theoretical study showed that for a large rectangular canal, \( \beta_0 \) was equal to 1, we decided to assume that this relation was still effective in the general case, which leaves only three parameters to identify for the dynamics: \( d = (\alpha_c \beta_c \alpha_d)^T \).

\[ \text{Figure 7: Criteria (dashed) and parameters vs number of iterations} \]

Identification: \( d = [0.415 \ 0.82 \ 334]^T, p = 0.5382 \). For variable withdrawals, we identified the parameters \( p_1 \) and \( p_2 \) of the periodic law (the periodicity is supposed to be known, derived from statistical studies on farmers behavior). The identification showed that \( p_1 \) and \( p_2 \) were very close the one to the other. The hypothesis of constant withdrawals is therefore validated.

4.3. Validation

To validate our results, we chose another period (25 to 30 August 1994), and used the identified dynamics to identify the withdrawals. Results are displayed in Figure 8.

\[ \text{Figure 8: Validation: input (dashed), measured output (solid) and model output (dotted)} \]

The withdrawal identified is \( p = 0.0956 \). The validation is considered to be satisfactory, as the prediction horizon needed for a good water management is of a few hours (the time delay of the reach). The knowledge based model is therefore accurate enough to simulate river flow when withdrawals are well identified. As they are almost constant during the irrigation season, as well as there are almost no precipitations, a control law based on this model is expected to be effective.

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5. CONCLUSION

We develop in this paper an approach to identify the dynamics and the withdrawals of an irrigation system described by a time-implicit finite-dimensional model based on simplified equations governing open-channel hydraulics. The identification is done with a two level relaxation algorithm. This approach enables to give to the manager two important parameters that are useful for the management of the system:

- the dynamics of the (nonlinear) system,
- the withdrawals.

The identification was not influenced by the daily variations of the withdrawals. The hypothesis of constant withdrawals can therefore be justified a posteriori. This hypothesis is nonetheless justified only for short periods (a few days) when the irrigation coefficient remain rather constant (no rain, or weather change, and in the middle of the irrigation season).

The identification procedure proposed will be extensively tested on rivers managed by the CACG, and results will be used for synthesis of robust controllers for variable discharges.

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