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Applied Mathematical Modelling 23 (1999) 829–846

APPLIED MATHEMATICAL MODELLING

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Robust continuous-time and discrete-time flow control of a dam–river system. (II) Controller design

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Received 20 August 1998; received in revised form 11 January 1999; accepted 4 March 1999

Abstract

The paper presents robust design methods for the automatic control of a dam–river system, where the action variable is the upstream flow rate and the controlled variable the downstream flow rate. The system is modeled with a linear model derived analytically from simplified partial derivative equations describing open-channel flow dynamics. Two control methods (pole placement and Smith predictor) are compared in terms of performance and robustness. The pole placement is done on the sampled model, whereas the Smith predictor is based on the continuous model. Robustness is estimated with the use of margins and also with the use of a bound on multiplicative uncertainty taking into account the model errors, due to the nonlinear dynamics of the system. Simulations are carried out on a nonlinear model of the river and performance and robustness of both controllers are compared to the ones of a continuous-time PID controller. © 1999 Elsevier Science Inc. All rights reserved.

Keywords: Open-channel irrigation system; Time delay; Robust control; Pole placement; Smith predictor; PID controller

1. Introduction

As water is becoming precious and rare, there is a growing interest for advanced management methods that prevent wastage of this vital resource. Irrigation is acknowledged as being the first water consumer in the world and many irrigation systems are still being managed manually, which leads to a low efficiency in terms of water delivered versus water taken from the resource. Automation is recognized as a possibly effective means to increase this efficiency [1].

One-dimensional open-channel flow dynamics are well represented by nonlinear partial differential equations (Saint–Venant equations), that are not easy to use directly for control design. Using linear approximated models, Papageorgiou et al. [2,3] and Sawadogo [4] already proposed design methods for dam–river open-channel systems, but they did not take into account robustness requirements, which are essential, especially for nonlinear systems controlled with linear regulators. As the process considered is dominated by long, varying time delays, the robustness to time delay variations is very important. Kosuth [5] studied the poles migration for varying time delays, but did not end with a reliable tuning method for robust control. Such a robust design

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approach is fairly recent for automatic control of irrigation systems. Only Corriga et al. [6], Jreij [7], Schuurmans [8] and Seatzu [9] have mentioned and evaluated model uncertainities. Their approach was restricted by the control of canal systems, where the elevation is controlled with intermediate gates, which is not the case for dam–river systems. Significant nonlinearities are encountered in the latter case, that make it compulsory to evaluate controller stability robustness.

The paper develops two classical SISO methods for the flow control of a dam–river system and compares their robustness to modeling errors, using the models obtained in the companion paper [10]: the nominal model is derived from simplified Saint–Venant equations and the robustness of the design is evaluated using the bound on multiplicative uncertainty, which captures possible variations in functioning points (i.e. different reference flow rates around which the process is linearized).

Firstly, a continuous-time Smith predictor controller is designed with a robust design method. Then, a discrete-time pole placement RST controller is designed, with a robust analysis. The pole placement is done on a sampled model of the system, because continuous-time pole placement methods cannot deal with infinite dimensional transfer functions as time-delays. A continuous-time PID controller is also designed and tuned following Haalman's rule, well suited for systems dominated by time-delays [11], to compare its performances to one of both the robust controllers.

The paper is organized as follows: Section 2 gives a description of the system and the design goals in terms of automatic control, Section 3 details the design methods, after a recall of classical robustness results. Section 4 shows the nonlinear simulation results, and some concluding remarks are given in Section 5.

The main contribution of the paper is the application and comparison of well-known robust control design and analysis methods to a nonlinear delay dominated dam–river system.

2. System description and design goals

2.1. Presentation of the system

The irrigation system considered the uses of natural rivers to convey water released from the upstream dam to consumption places. Farmers can pump water in the river when they need it without having to ask for it (it is an 'on demand' management).

The (simplified) system considered is depicted in Fig. 1, with a dam and one river reach with a measuring station at its downstream end, and a pumping station just upstream. Pumping stations are in fact distributed along the river. This is taken into account during the identification process, but for simplicity, it is supposed that all pumping stations can be aggregated into one at the end of

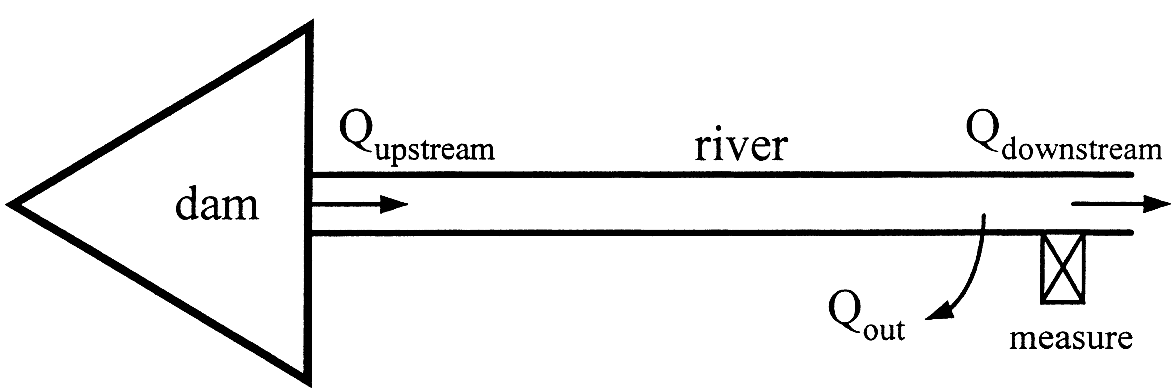

Fig. 1. Simplified dam–river system.

the reach. As this discharge Q_{out} is not measured and not controllable, it is considered as a perturbation, that has to be rejected.

The controlled variable is the flow rate at the downstream end of the river. The water elevation is not controlled, as the water distribution is done through pumping stations, not using gravity offtakes. The system is used mainly in summer for maize irrigation, when the flow rate is quite low. The control action variable is the upstream flow rate, and there is a local (slave) controller at the dam that acts on a gate such that the desired flow rate is delivered.

2.2. Control objectives

The objectives are twofold:

- satisfy the water demand from farmers (i.e. the discharge Q_{out});
- keep the flow rate at the downstream end of the reach close to a reference flow rate (target), defined for hygienic and ecological reasons.

The water demand can be predicted with a fairly good precision using weather forecast and soil-plant models. The variations in the demand are then considered as perturbations (due to unpredicted inflow or outflow), and have to be rejected by the controller.

Water demand predictions are used in an open loop controller, and a feedback controller is used to meet the second objective.

The main problem encountered in such systems is the possible instability due to varying time delays. To find a way to design a robust controller is therefore very interesting.

2.3. Modeling of the system and uncertainty description

The system is modeled by a second order plus delay transfer function:

$$F(s) = \frac{\exp(-s\tau)}{(1 + sK_1)(1 + sK_2)}.$$

The uncertainties due to different reference discharges are represented as an output multiplicative uncertainty. This multiplicative uncertainty captures time delay as well as dynamics variations, which are due to the nonlinearity of the process.

For a reference discharge $Q_0 \in [Q_{min}, Q_{max}]$, the transfer function $F(s)$ is written as

$$F(s) = [1 + \Delta_m(s)]F_0(s) \tag{1}$$

with $|\Delta_m(j\omega)| \leqslant l_m(\omega)\ \forall\omega$. $F_0(s)$ is the nominal model used to design the controller and l_m the bound on the multiplicative uncertainty Δ_m.

In Fig. 2, the input u corresponds to the upstream flow rate $Q_{upstream}$, the output y to the downstream flow rate $Q_{downstream}$ and w to the aggregated withdrawal Q_{out}.

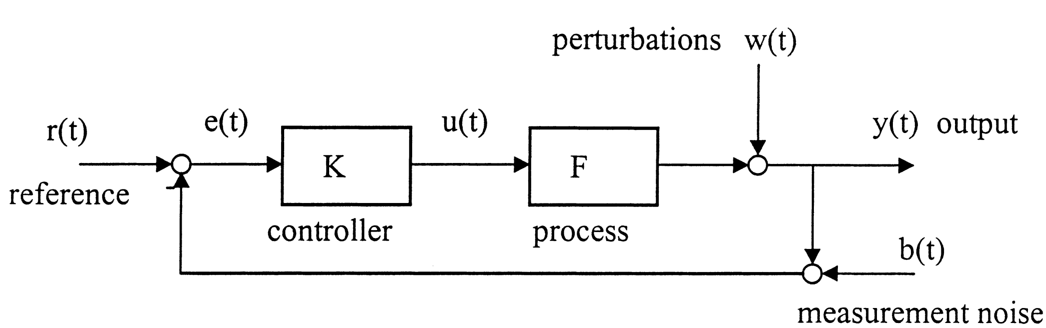

Fig. 2. Feedback system.

A discrete-time model $F^*(z)$ obtained by sampling of $F(s)$ with a zero order hold with a sampling period T_e will also be used, with corresponding multiplicative uncertainty (see Litrico and Georges [10] for the modeling part)

$$F^*(z) = z^{-r-1}\frac{c + dz^{-1} + ez^{-2}}{1 - az^{-1} + bz^{-2}}.$$

3. Robust control design

3.1. Robustness evaluation

Classical robustness measures given by gain and phase margins are not well suited for evaluating robustness to time delay variations. Modulus and delay margins [12] are more useful.

In the following, the definitions of classical robustness margins are recalled, along with simple explanations of their physical meaning. These margins offer a simple way to evaluate the robustness of a controlled system, in terms of acceptable variations in gain, phase or time delay.

3.1.1. Robustness margins

Consider the feedback system of Fig. 2. It gives the following relations:

$$y = S_y w + T_y(r - b), \qquad u = KS_y(r - b - w)$$

with

$$S_y = \frac{1}{1+L}, \quad T_y = \frac{L}{1+L}.$$

$L = FK$ is the open loop transfer function, S_y the output-perturbation sensitivity function, and T_y the complementary sensitivity function.

T_y and S_y are linked by the relation $S_y + T_y = 1$. The *modulus margin* M_m is defined as the minimal distance of the Nyquist plot of L to the point (−1, 0):

$$M_\mathrm{m} = \inf\{|1 + L(j\omega)|,\ \omega \in \boldsymbol{R}\}.$$

Then

$$M_\mathrm{m} = |1 + L(j\omega)|_{\min} = |S_y(j\omega)^{-1}|_{\min} = (|S_y(j\omega)|_{\max})^{-1} = \frac{1}{||Sy||_\infty},$$

where $||S_y||_\infty$ represents the maximum of $|S_y(j\omega)|$ for $\omega \in \boldsymbol{R}$.

A pure time delay τ introduces a phase lag $\omega\tau$ proportional to the frequency ω. The *delay margin* M_d is defined as the maximum of the time delays τ such that the feedback system is stable for a perturbed process $R_\tau F$ (R_τ represents the delay operator of transfer function $\mathrm{e}^{-\tau s}$):

$$M_\mathrm{d} = \frac{\varphi}{\omega_\mathrm{cr}},$$

where φ is the *phase margin* (in radians), and ω_cr the crossover frequency (in rad/s) where the Nyquist plot of L intersects the unit circle. These definitions are also extended to the case where the Nyquist plot intersects the unit circle at more than one point.

3.1.2. General robustness results for unstructured uncertainty

The robustness margins presented above only consider variations in gain, phase or time delay and not simultaneous variations. The use of unstructured uncertainty enables us to take into account the global modifications of the nominal transfer function. The Nyquist theorem gives general robustness results for such uncertainties.

With uncertainties represented in the multiplicative form, a condition of robust stability is [13]

$$|T_{y0}(j\omega)| = \left|\frac{L_0(j\omega)}{1+L_0(j\omega)}\right| < \frac{1}{|l_m(j\omega)|} \quad \forall\omega\in \boldsymbol{R} \tag{2}$$

with $L_0 = KF_0$, the nominal open loop transfer function.

This is a direct application of the Nyquist theorem, if the perturbed system has the same number of unstable poles as the nominal system.

For discrete-time systems, the same theorem applies, in the following form:

$$|T_{y0}(z)| = \left|\frac{L_0(z)}{1+L_0(z)}\right| < \frac{1}{|l_\mathrm{m}(z)|}, \quad z=\mathrm{e}^{j\theta}, \quad 0 \leqslant \theta \leqslant \pi.$$

3.2. Robust continuous Smith predictor design

3.2.1. Internal model control representation of the Smith predictor

The nominal transfer function F_0 is factored in two terms, F_{M0} being the part without delay

$$F_0(s) = F_{A0}(s)F_{M0}(s) \tag{3}$$

with

$$F_{A0}(s) = \exp(-s\tau_0) \text{ and } F_{M0}(s) = \frac{1}{(1+sK_{10})(1+sK_{20})}.$$

The classical Smith predictor is usually represented as in Fig. 3. Such a controller is interesting as, in the case of perfect modeling, the delay is eliminated from the closed loop equation [14]. The transfer from the reference r to the output y (which is the complementary sensitivity function T_y) is given by

$$T_y(s) = \frac{C(s)F(s)}{1+C(s)[F_{M0}(s) - F_{M0}(s)\exp(-s\tau_0) + F(s)]}.$$

If $F(s) = F_{M0}(s)\exp(-s\tau_0)$, it gives

$$T_{y0}(s) = \frac{C(s)F_{M0}(s)}{1+C(s)F_{M0}(s)}\exp(-s\tau_0).$$

It is then possible to design the controller $C(s)$ without taking the delay into account, as the characteristic polynomial does not depend on the delay.

To study the robustness of this feedback system, the controller is rewritten in the form of Internal Model Control [15] (see Fig. 4). The nominal complementary sensitivity function T_{y0} is then given by

$$T_{y0}(s) = Q(s)F_0(s).$$

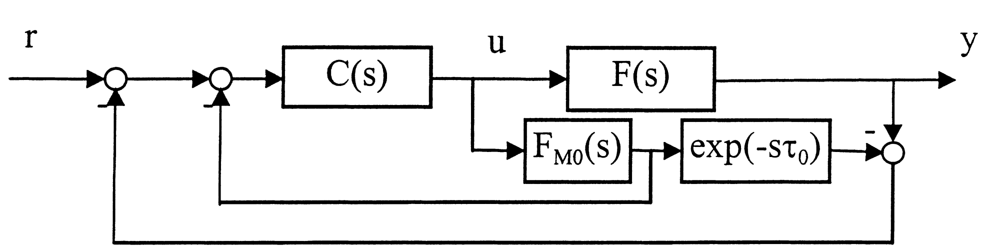

Fig. 3. Classical Smith predictor.

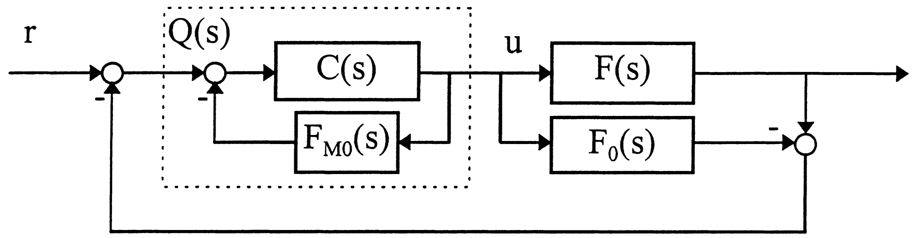

Fig. 4. IMC representation of the Smith predictor.

Q and C are linked by the following relation:

$$Q(s) = \frac{C(s)}{1 + C(s)F_{M0}(s)}.$$

3.2.2. Robust stability and performance of the Smith predictor

Robust stability: Using the IMC representation, the robust stability condition (2) becomes:

The system of Fig. 4 is stable for multiplicative uncertainties $|\Delta_{\mathrm{m}}(j\omega)| \leqslant l_{\mathrm{m}}(\omega)$ *if*:

- *the nominal system is stable*;
- $|Q(j\omega)F_0(j\omega)| < l_{\mathrm{m}}(\omega)^{-1}\ \forall\omega$.

Robust performance: Nominal performance is specified with an H_∞ constraint on nominal sensitivity function S_{y0} ($S_{y0} = 1 - T_{y0}$)

$$||S_{y0}(j\omega)w_2(j\omega)||_\infty < 1$$

or

$$|S_{y0}(j\omega)| < \frac{1}{|w_2(j\omega)|}\ \forall\omega,$$

where w_2 is a weighting function for performance.

For example, a choice of $w_2 = \mathrm{MP}^{-1}$, with MP a given positive real scalar ensures that the Maximum Peak of the modulus of the nominal sensitivity function S_{y0} stays below MP (see Laughlin et al. [13] for different choices of weighting functions w_2).

The performance is robust when the inequality is respected for $S_y(j\omega)$, i.e. for all models in the set described by F_0 and the bound on the multiplicative uncertainty l_{m}.

Robust stability and performance can be combined in one inequality

$$|T_{y0}(j\omega)|l_{\mathrm{m}}(\omega) + |[1 - T_{y0}(j\omega)]w_2(j\omega)| < 1 \quad \forall\omega$$

or

$$\frac{|T_{y0}(j\omega)|}{1 - |[1 - T_{y0}(j\omega)]w_2(j\omega)|} < l_{\mathrm{m}}(\omega)^{-1} \quad \forall\omega. \tag{4}$$

This condition can be checked by plotting frequency responses of both terms of the inequality [16].

The advantage of the IMC parametrization is that the design of the controller results in the choice of a single design parameter, as the H_2-optimal controller $Q(s)$ can be calculated analytically, and a filter $f(s)$ is added to ensure robustness to the feedback system

$$f(s) = \frac{1}{(1 + \lambda s)^n}$$

n is chosen to make $f(s)Q(s)$ proper, and λ is the design parameter of the IMC controller.

If $F(s) = F_{A0}(s)F_{M0}(s)$, the H_2-optimal controller Q minimizing the H_2 norm of the nominal error e for perturbations w, is given by [15]

$$Q = (F_{M0}w_M)^{-1}\{F_{A0}^{-1}w_A\}_*,$$

where operator $\{\sim\}_*$ means that after a partial fraction expansion, all terms containing the poles of F_{A0}^{-1} are omitted.

Robust performance design is done by trial and error on parameter λ.

1. For a given value of λ, check condition (4).
2. If Eq. (4) is not satisfied, increase λ.
3. Repeat 1 and 2 until Eq. (4) is satisfied.

If no value of λ lead to satisfaction of Eq. (4), the performance requirements have to be lowered.

3.2.3. Application

The system considered is a 20 km long river reach with the following physical parameters

$$\alpha_C = 0.415 \text{ m/s}; \quad \beta_C = 0.4; \quad \alpha_D = 332 \text{ m}^2/\text{s}; \quad \beta_D = 1.$$

The discharge is supposed to be inside the interval [0.5, 5] m^3/s.

It is represented by the nominal transfer function (3) with coefficients: $K_{10} = 9995.1 + 3310.5j$; $K_{20} = 9995.1 - 3310.5j$; $\tau_0 = 24463$ s, and the multiplicative uncertainty (1) bounded by $l_m(\omega)$.

The performance weighting function w_2 is chosen as

$$w_2(j\omega) = \frac{1}{\text{MP}}, \quad \text{with MP} = 3$$

which ensures that the maximum peak of $|S_y|$ stays below 3 for all models in the set described by the nominal model and the multiplicative uncertainty.

For $F_0(s)$ given by

$$F_0(s) = F_{A0}(s)F_{M0}(s),$$

a step perturbation input $w(s) = 1/s$ ($w_M = 1/s$, $w_A = 1$), the optimal controller Q is given by

$$Q(s) = (F_{M0}w_M)^{-1}\{F_{A0}^{-1}w_A\}_* = (F_{M0}(s)/s)^{-1}\{\exp(s\tau_0)/s\}_*$$

or

$$Q(s) = (1 + sK_{10})(1 + sK_{20}).$$

As proposed by Laughlin et al. [13] the initial value for filter coefficient iterations is given by

$$\lambda_0 = \left[\left(\frac{\text{MP}+1}{\text{MP}-1}\right)^2 - 1\right]^{1/2} \Big/ \omega',$$

where ω' is the frequency for which $l_m(\omega') = 1$.

The final value obtained is $\lambda = 18577 \text{ s} = 0.59\lambda_0$, for which the robust performance condition is satisfied.

Figs. 5 and 6 show that the robust performance and robust stability conditions are satisfied.

The robustness margins obtained for the three controllers are given in Table 1.

As we can check from Fig. 7, the sensitivity functions for the nominal and the uncertain systems are below 3, which was the maximum peak (MP) allowed as a robust performance requirement.

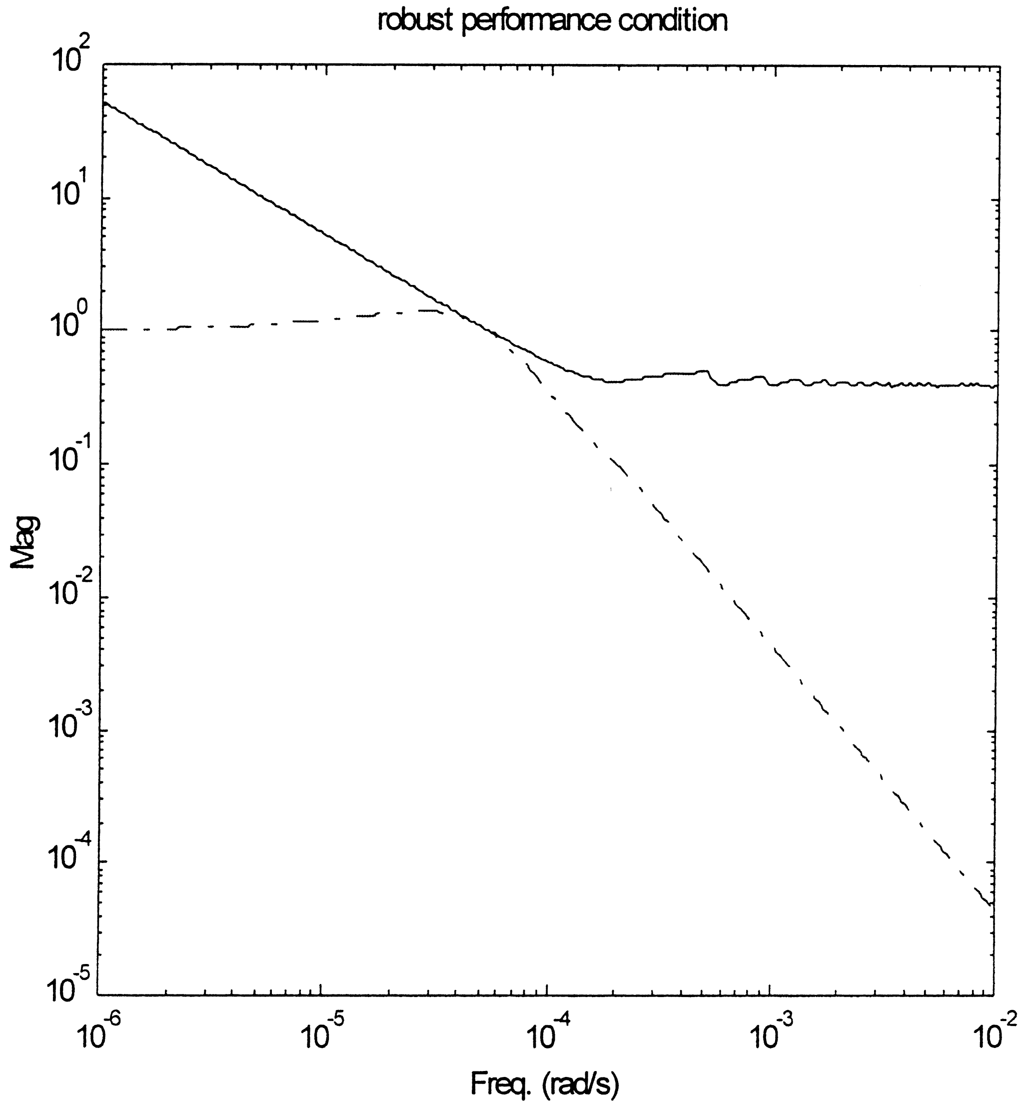

Fig. 5. Frequency responses of $l_m(\omega)^{-1}$ (continuous line) and $|T_{y0}(j\omega)|/(1-|[1-T_{y0}(j\omega)]w_2(j\omega)|)$ (dashed), SP controller.

3.3. Robust digital pole placement design

3.3.1. Digital RST controller

With the transfer function of the system written as

$$F^*(z) = z^{-r-1}\frac{c + dz^{-1} + ez^{-2}}{1 - az^{-1} + bz^{-2}} = \frac{B(z)}{A(z)},$$

the RST regulator is represented in Fig. 8, where R, S and T are polynomials of z.

The output y is expressed as function of w, the output disturbance, y_c the targeted output, and b, the measurement noise

$$y = \frac{AS}{AS + BR}w + \frac{BT}{AS + BR}y_c - \frac{BR}{AS + BR}b,$$

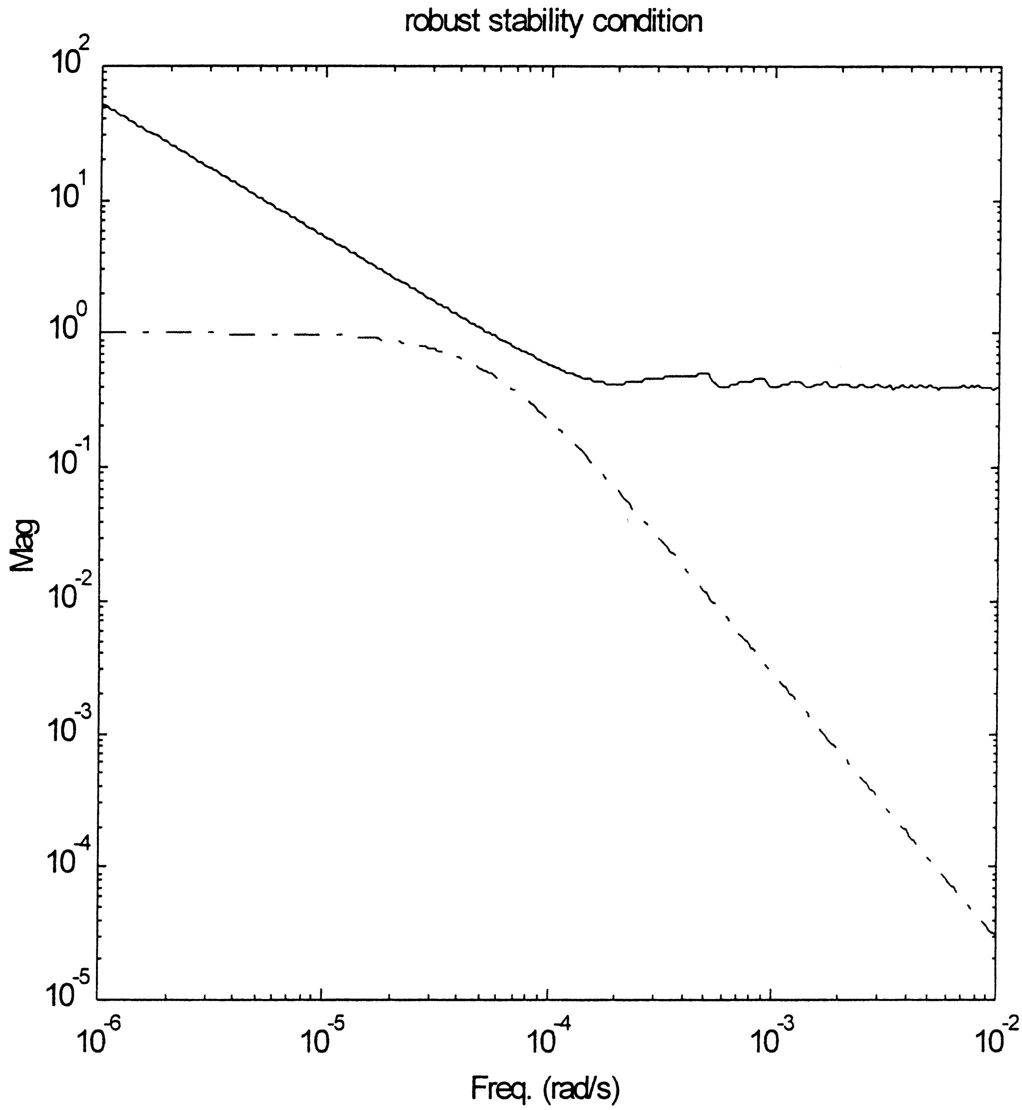

Fig. 6. Frequency responses of $l_m(\omega)^{-1}$ (continuous) and $|T_{y0}(j\omega)|$ (dashed), SP controller.

Table 1
Robustness margins for the three controllers

Controllers	SP	RST	PID
Gain margin (M_g)	3.24	3.49	2.36
Delay margin (M_d)	19.6 h	16.4 h	9.2 h
Advance margin (M_a)	86.8 h	69.8 h	54.8 h
Modulus margin (M_m)	0.66	0.69	0.52

(argument z is omitted for readability).

Sensitivity functions are expressed as functions of polynomials R, S, A and B

$$S_y = \frac{AS}{AS + BR},$$

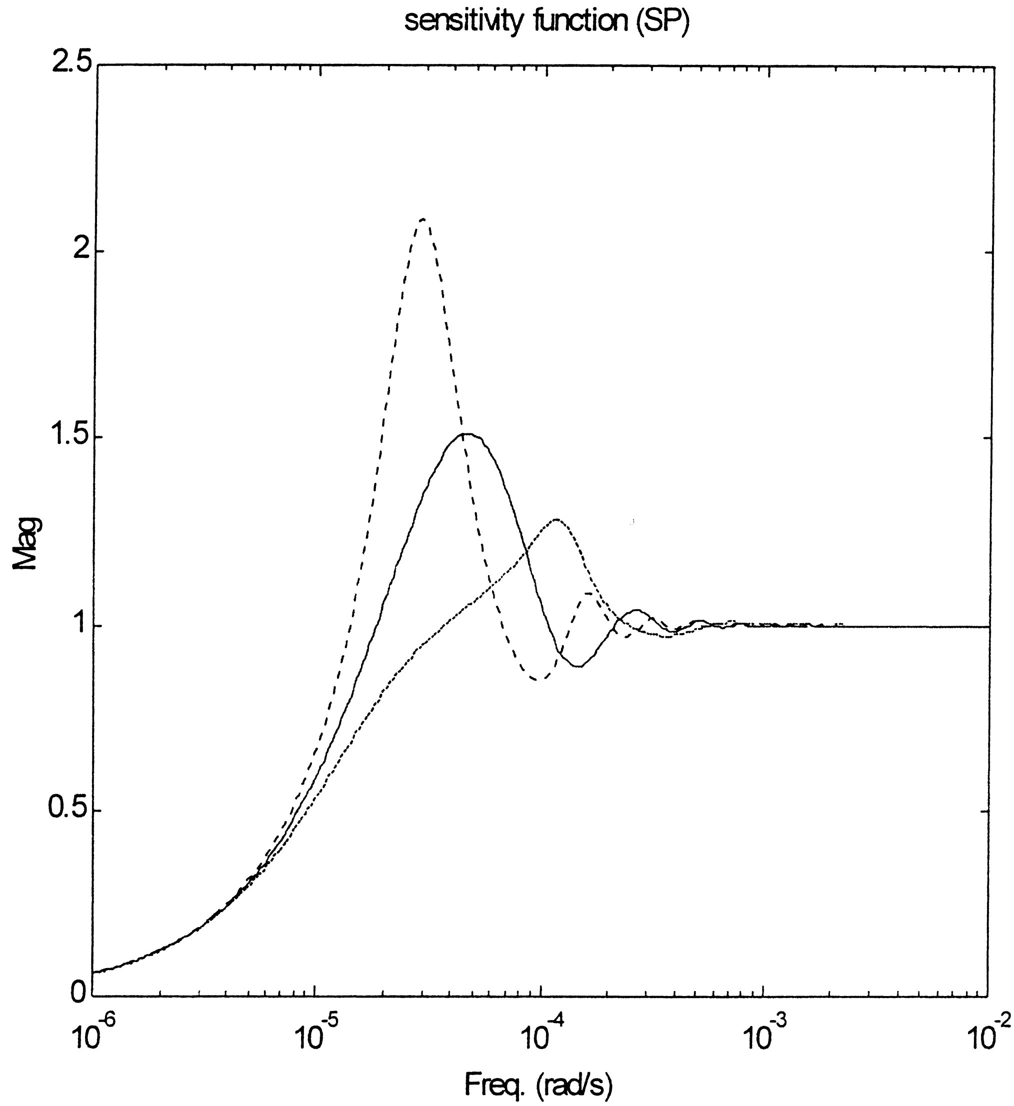

Fig. 7. Sensitivity functions for nominal system and perturbed systems (dashed -5 m^3/s- and dotted -0.5 m^3/s-lines), SP controller.

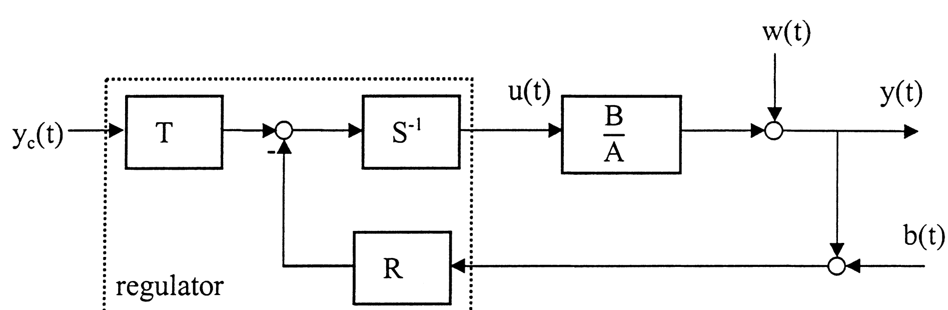

Fig. 8. Structure of the 'RST' regulator.

$$T_y = \frac{BR}{AS + BR}.$$

The open loop transfer function is

$$L = \frac{BR}{AS}.$$

Stability of the closed loop system is ensured if the roots of $AS + BR$ are inside the unit circle.

3.3.2. Robust pole placement [17]

The method provides a way to place poles of a feedback system with two design parameters, a control horizon T_c and a filtering horizon T_f. It is based on the following heuristic remark, valid for a stable process: the less the poles of the process are modified by the feedback, the more robust the closed loop system is with respect to model uncertainties.

The method proposes to place the poles of the feedback system from the poles of the nominal process:

- In continuous-time the poles of the process which are to the right of a straight line $x = -1/T_c$ are projected on this line.
- In discrete-time, the poles of the nominal system with modulus greater than $R_c = \exp(-T_e/T_c)$ are projected on the circle of radius R_c, the others are left in place.

This gives the n dominant poles (n is the degree of $A(z^{-1})$), and the $n+1$ filtering poles are determined with another parameter T_f and a real pole is added at $\exp(-T_e/T_f)$.

The closed loop polynomial has $2n+1$ zeros, determined with the two design parameters T_c and T_f.

In order to reject constant perturbations, an integrator is included in the regulator, which means that $S(z)$ can be factorized in

$$S(z) = (z-1)S'(z).$$

The pole placement problem is to find polynomials R and S', such that

$$A(z)(z-1)S'(z) + B(z)R(z) = P(z).$$

$P(z)$ being the desired characteristic polynomial (with roots at the desired locations) for the closed loop system.

This Bezout equation can be written as a linear system in the polynomial coefficients of R and S' and solved using linear systems resolution methods [18].

3.3.3. Application

The process considered is the same river reach as in Section 3.2.3, sampled with a sampling period $T_e = 3600$ s. The command horizon is $T_c = 1 \times T_e$, and the filtering horizon $T_f = 7 \times T_e$.

As the discrete-time process is obtained by sampling a continuous-time process, aliasing is encountered for frequencies greater than the Nyquist frequency $\omega_N = \pi/T_e$ (here $\omega_N = 8.7266\ 10^{-4}$ rad/s).

The RST controller is robust stable with respect to multiplicative uncertainties, but does not satisfy the robust performance condition, which is slightly violated (Fig. 9). Nonetheless, the sensitivity functions for models corresponding to extreme flow rates are below 3 (Fig. 10). This is due to the fact that the uncertainties described by our multiplicative bound are overestimated compared to real uncertainties due to variable reference discharges; the set of models described by the nominal model and a multiplicative uncertainty bounded by l_m is larger than the set of models described by different reference discharges. As we take into account more uncertainties than necessary, we end with a slightly too restrictive condition.

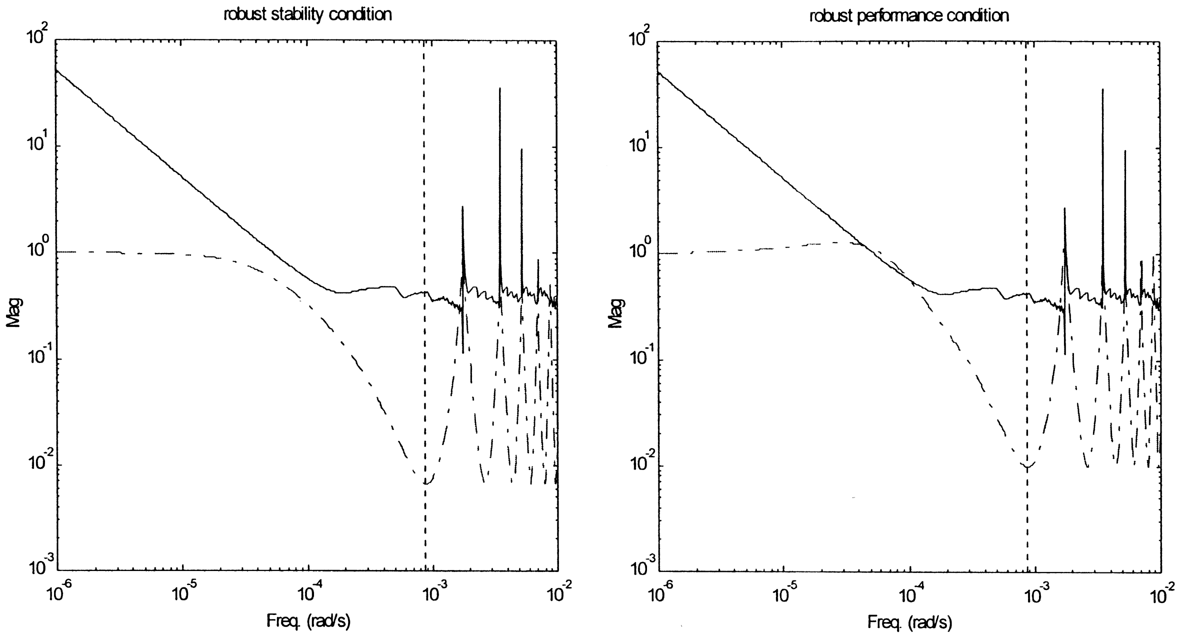

Fig. 9. Robust stability and robust performance conditions, RST controller.

It should also be noticed that the modulus margin of the RST controller is better than the one of the SP controller, whereas the SP controller is indeed more robust than the RST. This is due to the fact that the classical margins only take into account the nominal open loop transfer function and not the model uncertainties. This is why the graphs provided also show the perturbed systems, corresponding to extreme flow rates.

3.4. PID controller design

3.4.1. Haalman's method

Both controllers are compared to a continuous-time PID controller obtained with Haalman's method, suited for systems where dynamics are dominated by dead-time [11]. This is a loop shaping method where the desired loop transfer function L_0 is specified, and the controller transfer function is obtained as

$$K = \frac{L_0}{F},$$

where F is the system transfer function.

Such an approach can give PID controllers provided that L_0 and F are sufficiently simple.

For systems with a time delay τ, Haalman has suggested to choose

$$L_0(s) = \frac{2}{3\tau s} e^{-s\tau}.$$

The value 2/3 was found by minimizing the mean square error for a step change in the set point. This choice gives a sensitivity MP = 1.9 (or a modulus margin $M_m = 1/1.9 = 0.53$), and a delay margin

$$M_d = \frac{3\pi - 4}{4}\tau = 1.36\,\tau.$$

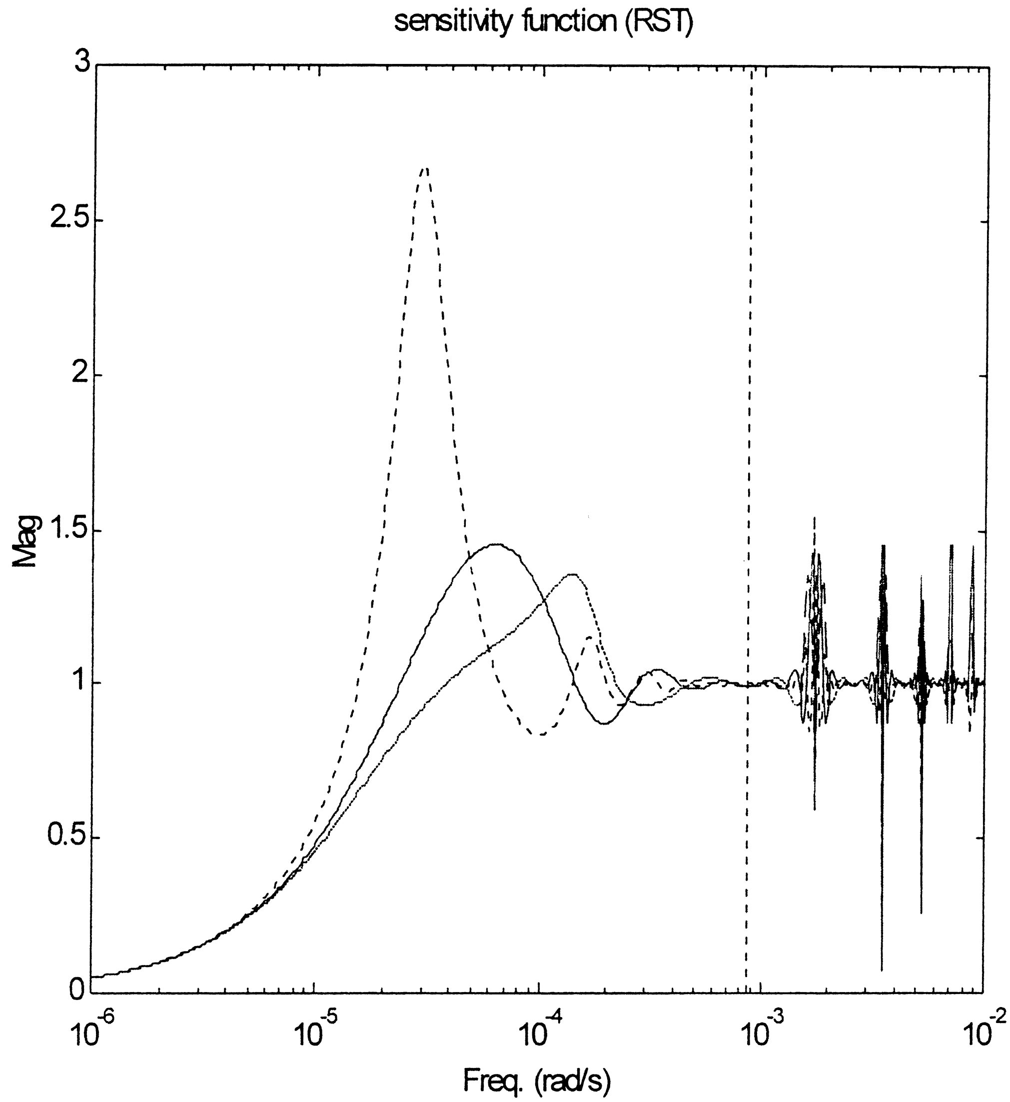

Fig. 10. Sensitivity functions for nominal and perturbed systems (dashed -5 m^3/s- and dotted -0.5 m^3/s-lines), RST controller.

For systems with the transfer function

$$F_0(s) = \frac{\exp(-s\tau_0)}{(1 + sK_{10})(1 + sK_{20})},$$

the controller obtained is a PID

$$K(s) = K_p\left(1 + \frac{1}{sT_i} + sT_d\right)$$

with the parameters

$$K_p = \frac{2(K_{10} + K_{20})}{3\tau_0},$$

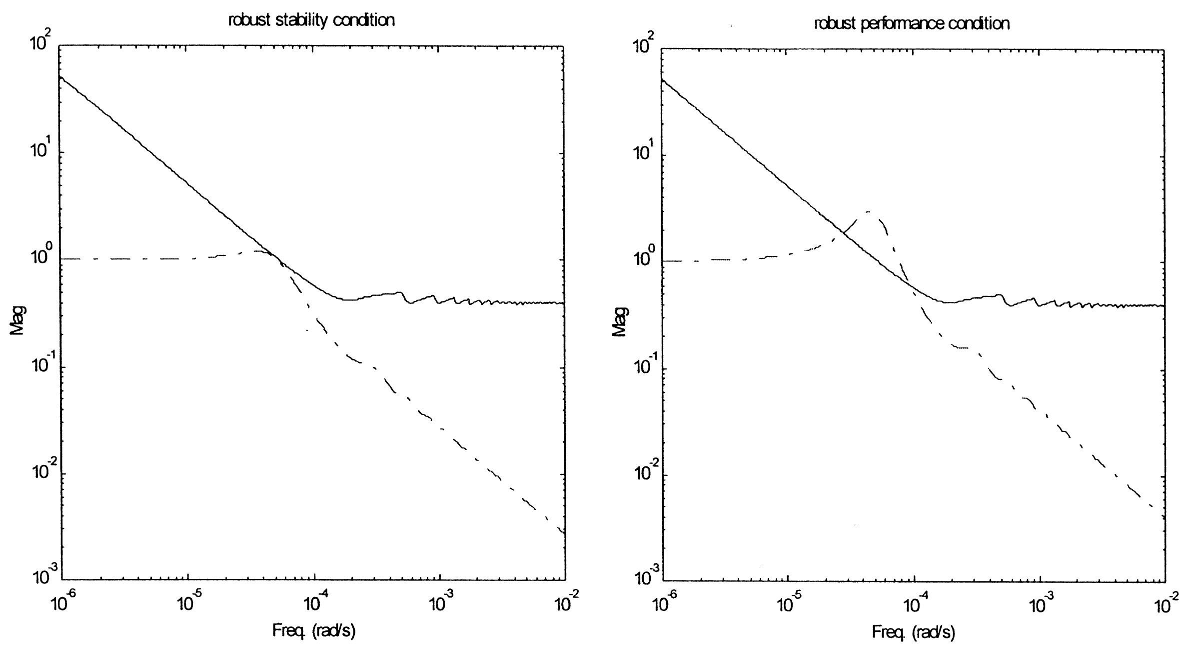

Fig. 11. Robust stability and robust performance conditions, PID controller.

$$T_i = K_{10} + K_{20},$$

$$T_d = \frac{K_{10}K_{20}}{K_{10} + K_{20}}.$$

Such a controller cancels the poles of the system, which may lead to controllability problems if the process is lag dominated, which is usually not the case in dam–river systems.

3.4.2. Application

With the river considered, the following coefficients are obtained:

$$K_p = 0.5448,$$

$$T_i = 19990 \text{ s},$$

$$T_d = 5546 \text{ s}.$$

The PID controller is robust stable with respect to multiplicative uncertainties, but does not satisfy the robust performance condition (Fig. 11).

Its robustness margins are rather good (cf. Table 1), but are not representative of its real robustness, as already mentioned. Its robustness is better evaluated by looking at Fig. 12.

4. Results of simulations and discussion

4.1. Simulations

Results of simulations given by both controllers are compared to those given by a continuous-time PID controller, tuned following Haalman's method. Simulations are done on a nonlinear

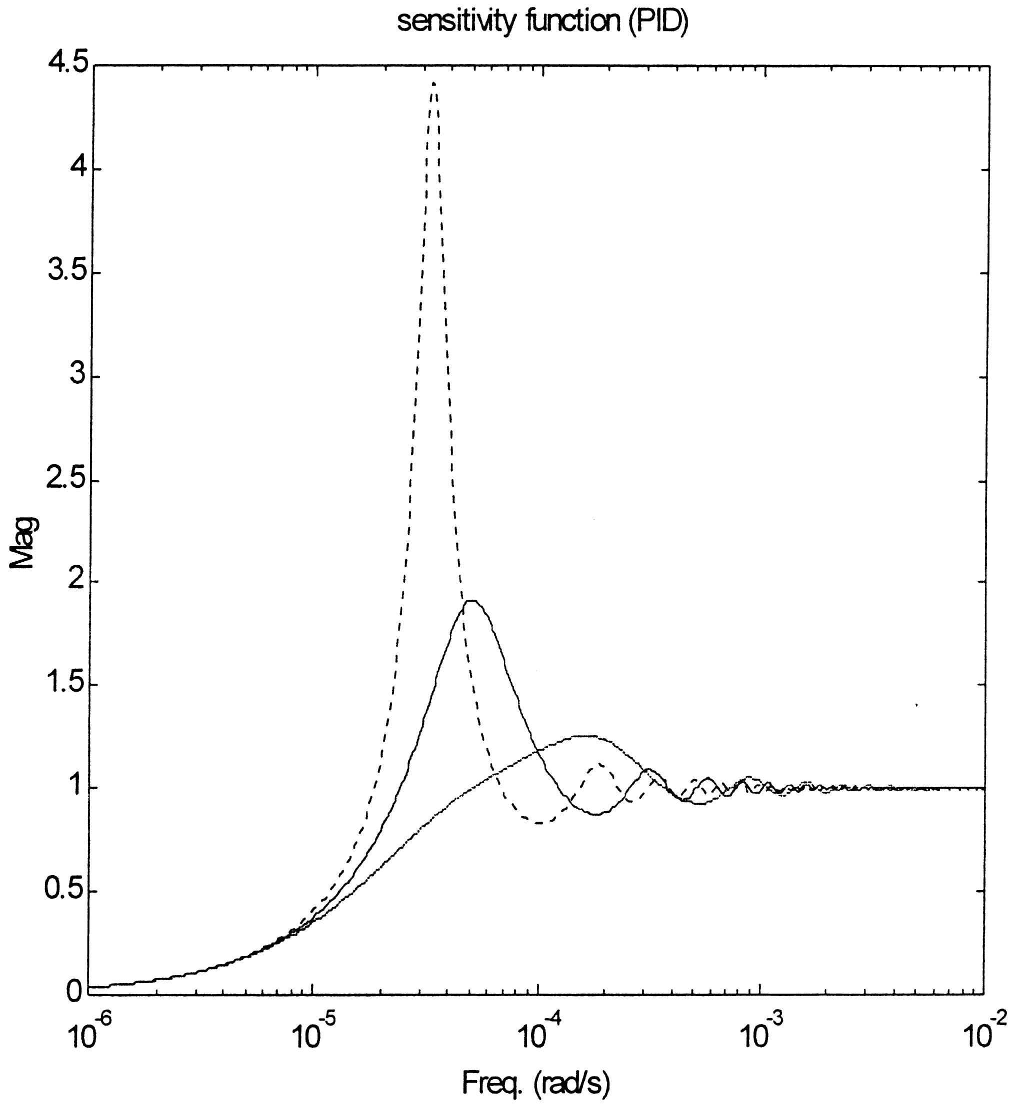

Fig. 12. Sensitivity functions for nominal (continuous line) and perturbed systems (dashed -5 m^3/s- and dotted -0.5 m^3/s-lines), PID controller.

model. The simulation scenario is an unknown disturbance rejection: an unpredicted withdrawal of 0.5 m^3/s occurs at time $t = 30$ h. The simulations are performed around an average flow rate (2 m^3/s) and around a low flow rate (0.5 m^3/s). Results are presented for the three controllers with the command signal and the output signal, for the Smith Predictor (SP), the pole placement controller (RST) and the PID controller from left to right in Figs. 13 and 14.

4.2. Discussion

Results of simulation show that all controllers are stable, as suggested by the robustness analysis in the frequency domain. It should be mentioned that we have no theoretical insurance for this, as the stability is ensured by Eq. (2) for a set of *linear* models captured in the nominal model and the multiplicative uncertainty, but not for the *nonlinear* system.

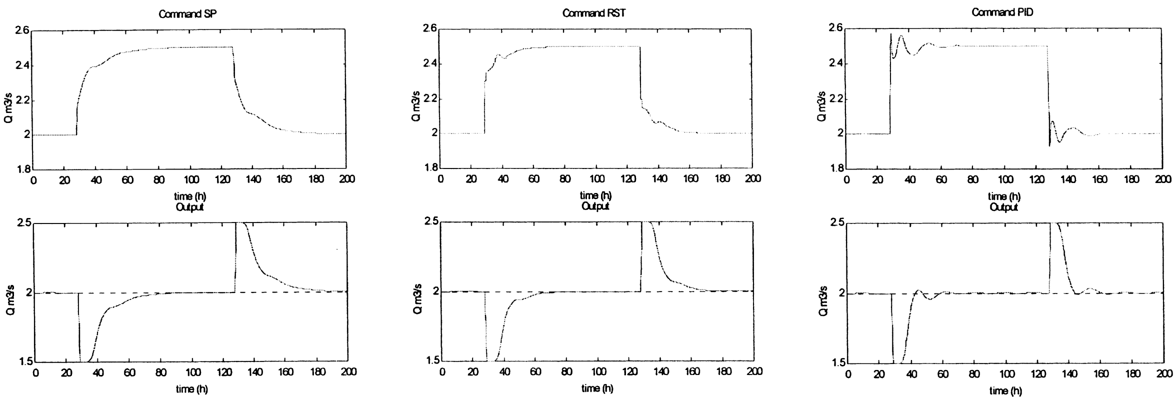

Fig. 13. Unknown disturbance rejection around $Q_0 = 2$ m^3/s.

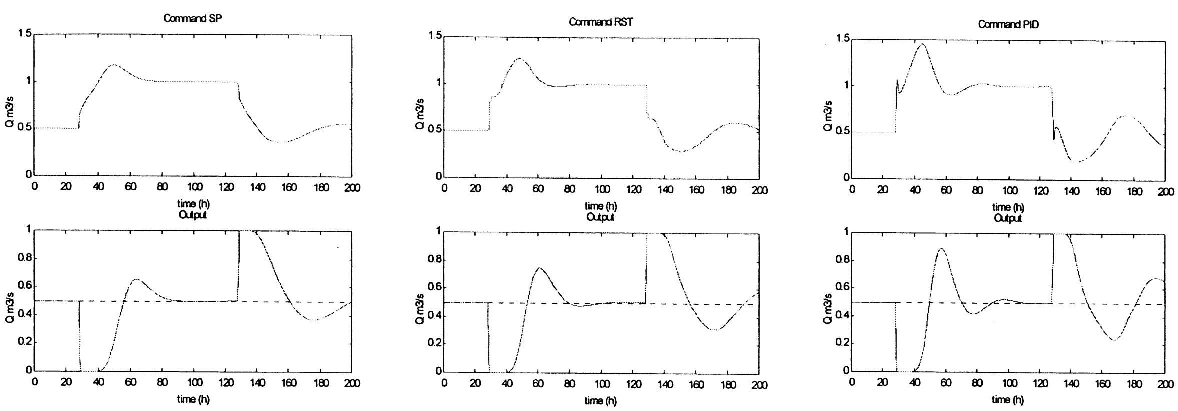

Fig. 14. Unknown disturbance rejection around $Q_0 = 0.5$ m^3/s.

The PID controller is the quicker to respond to unpredicted disturbances, but it is also the more oscillating. As such oscillating commands would not be allowed in reality, it would have to be filtered, therefore leading to slower responses. The overshoot in reaction of the withdrawal around 0.5 m^3/s is not acceptable, as one important objective is to save water and such reaction would clearly lead to a waste of water.

The RST controller behaves well, although oscillating around $Q_0 = 0.5$ m^3/s. The Smith Predictor is a little slower than the other two, but gives realistic commands for all operating points.

It is important to notice the fact that there is a trade-off between robustness and performance. A certain level of performance cannot be achieved without losing some robustness and respectively, to increase the robustness of a feedback loop, the performance requirements must be lowered. The same dilemma may occur in nonlinear control design systems, but a robust nonlinear controller should be more performing than a linear one, keeping the same robustness requirements.

More than the results of simulation, where different values of tuning parameters can be chosen depending on the requirements, the emphasis is made on the tuning methods, and the robustness analysis tools developed.

Both tuning methods are easy to manage and robustness can be checked a posteriori. The tuning parameters have a clear physical interpretation:

- The filter parameter λ in the IMC design is called the robustness filter: the larger λ, the more robust the controlled system, but perturbations are rejected slowly.
- The robust pole placement method has two tuning parameters T_c and T_f; T_c is a control horizon and T_f a filtering horizon. The larger they are, the slower the controlled system will be. Their values can be adjusted by trial and error, using the pole location, or the value of robustness margins as a tuning criteria.

On the other hand, the PID controller does not offer clear tuning parameters and the method may not be suited for other systems.

5. Conclusion

The paper presents some tools to check the robustness of feedback loops. The tools are illustrated on two different approaches:

- *an a priori approach to design a robust continuous-time Smith predictor*: the controller is designed taking into account the multiplicative uncertainty on the nominal model;
- *an a posteriori approach to design a robust discrete-time pole placement RST controller*: the design parameters T_c and T_f are tuned by trial and error until satisfactory robustness margins are obtained.

Controllers proved to be robust and have a good performance, compared to a PID controller tuned with Haalman's method and both methods are suited for real-time applications.

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